

FUNCTIONAL CORRELATION, DESIGN INFORMATION ENTROPY AND THE DEPENDENCY OF AXIOMATIC DESIGN AXIOMS



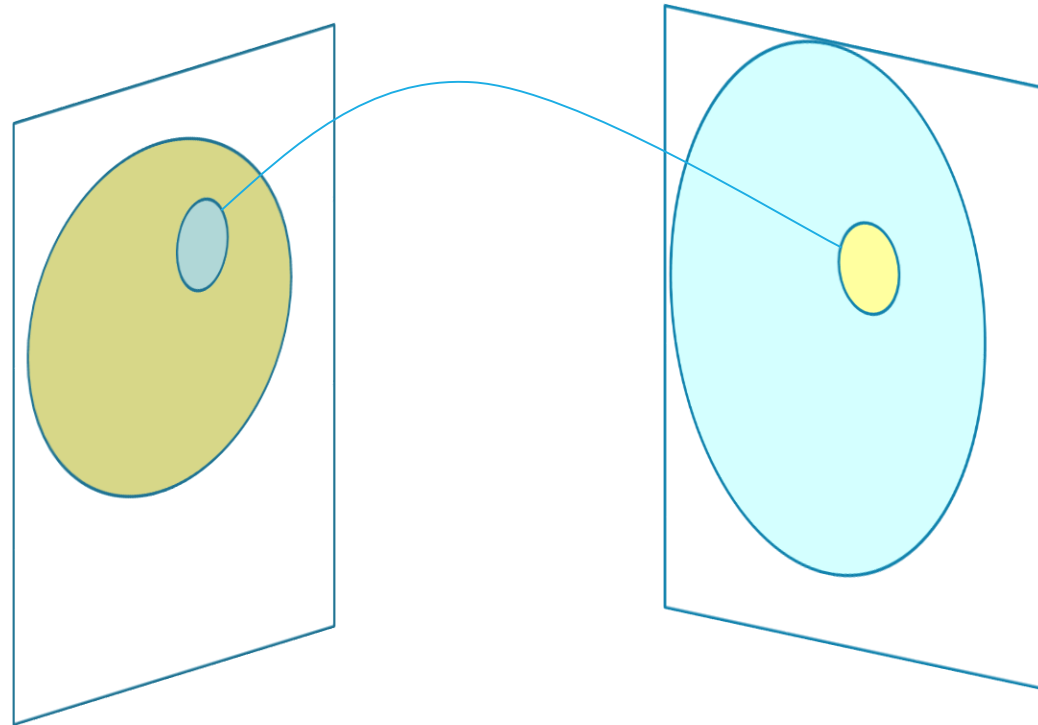
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AXIOMATIC DESIGN



Generalised parameter space x
(*Design parameters, DP*)

Functional space y
(*Functional requirements, FR*)

AXIOMATIC DESIGN

In Axiomatic Design, AD, was introduced by Nam P Suh. It has been used in a wide range of applications and in different ways.

A central concept in Axiomatic design is the design matrix that represents the relationship between the design parameters, x_D and the functional requirements f_R . The relationship can be written as:

Example:

$$f_R = A \times x_D$$

$$\begin{pmatrix} f_{R1} \\ f_{R2} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_{D1} \\ x_{D2} \end{pmatrix}$$

AXIOMATIC DESIGN

Developed by Namh P. Suh, MIT.:

Axioms are truths that cannot be derived but for which there are no counter examples.

Scientific theory should be based on axioms.

AXIOMATIC DESIGN: TWO (2) AXIOMS

First axiom: The independence axiom

Second axiom: The information axiom

- The design with the least information is the best.
(The most robust solution is the best)

THE INDEPENDENCE AXIOM

Uncoupled design

$$\begin{pmatrix} f_{R1} \\ f_{R2} \end{pmatrix} = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} \begin{pmatrix} x_{D1} \\ x_{D2} \end{pmatrix}$$

Coupled design

$$\begin{pmatrix} f_{R1} \\ f_{R2} \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} x_{D1} \\ x_{D2} \end{pmatrix}$$

EXAMPLE: AIRPLANE (FIXED WING)



$$\begin{pmatrix} \text{lift} \\ \text{thrust} \end{pmatrix} = \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix} \begin{pmatrix} \text{wing} \\ \text{propulsion} \end{pmatrix}$$

Uncoupled design

EXAMPLE: ORNITHOPTER (AND HELICOPTER)

$$\begin{pmatrix} \text{lift} \\ \text{thrust} \end{pmatrix} = \begin{pmatrix} X & X \\ X & X \end{pmatrix} \begin{pmatrix} \text{wing} \\ \text{propulsion} \end{pmatrix}$$

Coupled design



SENSITIVITY ANALYSIS

Linearization around a design point:

$$\mathbf{y}_0 + \Delta\mathbf{y}_0 = \Delta\mathbf{x}\mathbf{K} + \mathbf{f}(\mathbf{x}_0)$$

where \mathbf{K} is the Jacobian, where the elements are defined as:

$$k_{ij} = \frac{\partial y_i}{\partial x_j}$$

This is hence an analytical representation of the design matrix \mathbf{A} .

Normalized sensitivities (non-dimensional):

$$k_{ij}^0 = \frac{x_j}{y_i} \frac{\partial y_i}{\partial x_j}$$

SENSITIVITY ANALYSIS

EXAMPLE: ELECTRIC MOTORCYCLE

Functional requirements

- R (at constant speed 70 km/h) and acceleration time, t_a (0-70km/h)

design parameters

- battery size, m_b , and engine power, P_m .



SENSITIVITY ANALYSIS

ELECTRIC MOTORCYCLE RANGE

The range can be under some assumptions (only air resistance and constant speed) calculated as:

$$R = \frac{2k_b m_b \eta}{C_d A_0 \rho v^2}$$

Here:

- k_b is the battery energy density,
- m_b is the mass of the battery.
- η is the combined efficiency of battery and motor.
- C_d is the aerodynamic drag coefficient.
- A_0 is the frontal area and v being the vehicle speed.



SENSITIVITY ANALYSIS: ELECTRIC MOTORCYCLE

The acceleration time can be calculated as (assuming no air and rolling resistance, and constant power independent of speed)

$$t_a = \frac{mv^2}{2P\eta_a}$$

- Where the total weight is: $m = m_0 + m_b$.

The design relation matrix is obtained from:

$$\begin{pmatrix} R \\ t_a \end{pmatrix} = K \times \begin{pmatrix} m_b \\ P_m \end{pmatrix}$$



SENSITIVITY ANALYSIS: ELECTRIC MOTORCYCLE

The normalized sensitivity matrix K^0 can be calculated as:

$$K^0 = \begin{pmatrix} \frac{m_b}{R} \frac{\partial R}{\partial m_b} & \frac{P_m}{R} \frac{\partial R}{\partial P_m} \\ \frac{m_b}{t_a} \frac{\partial t_a}{\partial m_b} & \frac{P_m}{R t_a} \frac{\partial t_a}{\partial P_m} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \sigma_m & -1 \end{pmatrix}$$

where $\sigma_m = \frac{m_b}{m_0 + m_b}$

Hence: $\begin{pmatrix} \Delta R/R \\ \Delta t_a/t_a \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \sigma_m & -1 \end{pmatrix} \times \begin{pmatrix} \Delta m_b/m_b \\ \Delta P_m/P_m \end{pmatrix}$



NORMALIZED AIRCRAFT DESIGN SENSITIVITY



System characteristics	Units	Actual value	θ	C_r	taper	t_c	lambda	x_w	Bht	We	T	W_f	V_{cruise}
			18.83	4.07	0.57	0.07	0.18	6.77	10.28	37522.49	17255.22	28968.08	185.83
Range	km	6120.51	0.16	-0.39	-0.18	-0.17	0.01	0.05	-0.27	-0.05	12.66	5.30	0.00
Payload	N	7473.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Liftoff distance	m	473.34	-1.29	-0.72	-0.28	0.00	0.00	0.00	0.00	1.09	-0.91	0.84	0.00
Landing distance	m	264.57	-0.55	-1.50	-0.66	0.00	0.00	0.00	0.00	0.47	0.18	0.36	0.00
Takeoff weight	N	80242.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.18	0.36	0.00
Required weight quotient		1.00	0.03	-0.03	-0.01	-0.03	0.00	0.00	0.00	-0.49	0.15	0.29	0.00
Buffeting speed quotient		0.63	-0.50	-0.51	-0.21	0.00	0.02	0.00	0.00	0.23	0.09	0.18	-1.01
Optimum cruise speed	m/s	177.52	-0.63	-0.13	-0.04	-0.05	0.00	0.00	-0.11	0.23	0.09	0.18	0.00
Landing speed	m/s	34.93	-0.50	-0.51	-0.21	0.00	0.00	0.00	0.00	0.23	0.09	0.18	0.00
Liftoff speed	m/s	38.63	-0.50	-0.51	-0.21	0.00	0.00	0.00	0.00	0.23	0.09	0.18	0.00
Stall speed	m/s	76.89	-0.50	-0.51	-0.21	0.00	0.00	0.00	0.00	0.23	0.09	0.18	0.00
Emissions		0.62	-0.16	0.39	0.18	0.16	-0.01	-0.05	0.27	0.05	-12.67	-4.31	0.00
Consumption	1/km	0.00	-0.16	0.39	0.18	0.16	-0.01	-0.05	0.27	0.05	-12.67	-4.31	0.00
Rotation	m	10.94	-2.19	0.00	0.00	0.00	0.00	-1.62	5.95	0.00	0.00	0.00	0.00
Stability		1.53	-1.52	0.00	0.00	0.00	0.00	4.22	3.48	0.00	0.00	0.00	0.00
Cost	kEUR	51964.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.29	0.00	0.00

DESIGN INFORMATION ENTROPY

The second axiom in axiomatic design is regarding the minimization of information.

Information theory as introduced by Shannon [9], provides a framework for quantitatively describe information content in general. For the case of continuous variables, it can be written

$$H_c = - \int_{-\infty}^{\infty} p(x) \log_2(p(x)) dx$$

This gives a measure of the average information content of a variable x . Here $p(x)$ is the probability density function.

DESIGN INFORMATION ENTROPY

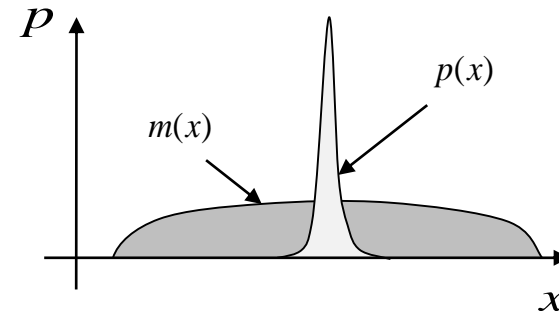
One problem with this expression is that it does not make sense unless x is non-dimensional, since the probability density function has the unit of the inverse of x .

Introducing the Kullback-Leibler divergence

$$H_{rel} = - \int_{-\infty}^{\infty} p(x) \log_2 \left(\frac{p(x)}{m(x)} \right) dx$$

Generalized

$$H_{rel} = - \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p(x_1 \dots x_n) \log_2 \left(\frac{p(x_1 \dots x_n)}{m(x_1 \dots x_n)} \right) dx_1 \cdots dx_n$$



DESIGN INFORMATION ENTROPY

A rectangular distribution of $m(x)$ in the bounded interval $x \in [x_{min}, x_{max}]$, with $x_R = x_{max} - x_{min}$ would mean that the distribution of the design space is a space of equal possibilities. For the rectangular probability distributions this can simply be written as:

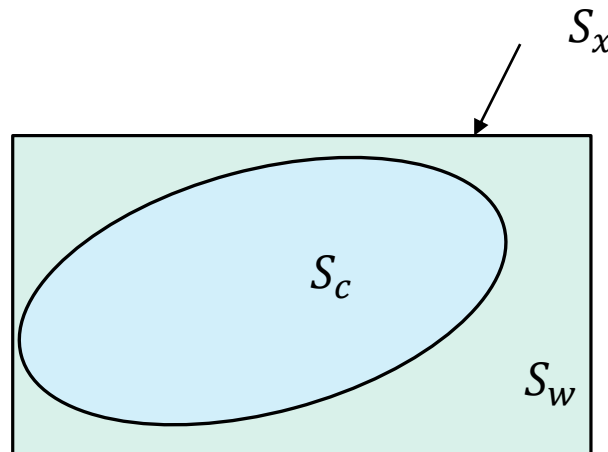
$$I_x = \log_2 \frac{S_1}{S_2}$$

where S_1 could be the design space and S_2 e.g. the region of the design space that fulfills the requirement range.

DESIGN INFORMATION ENTROPY

WASTED DESIGN SPACE

Hence, the amount of information needed to define a design relative to a design space can be calculated. According to this the part of the design range S_x that falls outside of S_c is here called S_w .



FUNCTIONAL CORRELATION

The normalized sensitivity matrix K^0 can be calculated as:

$$\begin{pmatrix} \Delta R/R \\ \Delta t_a/t_a \end{pmatrix} = K^0 \times \begin{pmatrix} \Delta m_b/m_b \\ \Delta P_m/P_m \end{pmatrix}$$

Hence:
$$\begin{pmatrix} \Delta R/R \\ \Delta t_a/t_a \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \sigma_m & -1 \end{pmatrix} \times \begin{pmatrix} \Delta m_b/m_b \\ \Delta P_m/P_m \end{pmatrix}$$

FUNCTIONAL CORRELATION

A measure of the angle between two vectors

The elements of the correlation matrix are calculated as.

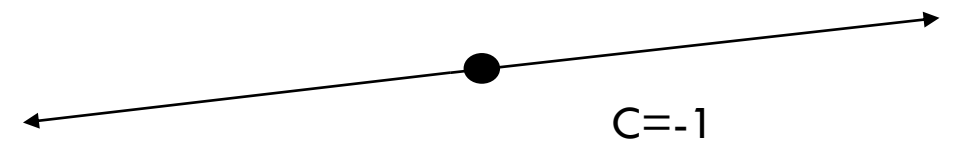
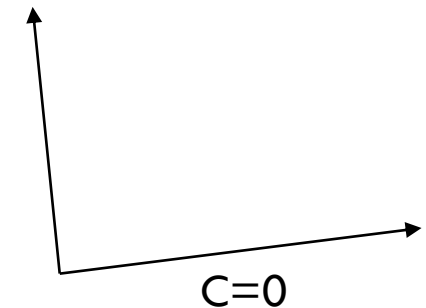
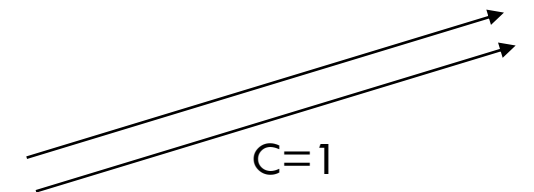
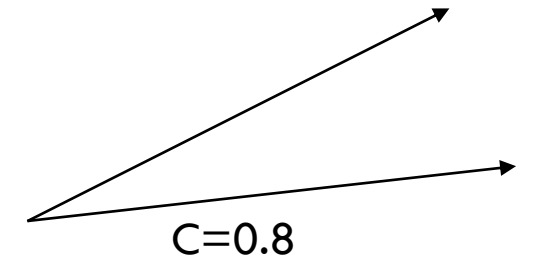
$$c_{ik} = \frac{\frac{1}{n} \sum_{j=1}^n k_{ij}^0 k_{kj}^0}{s_i s_k}$$

Where

$$s_i = \sqrt{\frac{1}{n} \sum_{j=1}^n (k_{ij}^0)^2}$$

With $m_b = m_0/4$ the correlation matrix for the electric motorcycle becomes:

$$C = \begin{pmatrix} 1 & 0.196 \\ 0.196 & 1 \end{pmatrix}$$



AIRCRAFT FUNCTIONAL CORRELATIONS (ADJUSTED)



System Characteristics		Range	Payload	Liftoff distance	Landing distance	Takeoff weight	Required weight quotient	Buffeting speed quotient	Optimum cruise speed	Landing speed	Liftoff speed	Stall speed	Emissions	Consumption	Rotation	Stability	Cost
		6120.51	7473.00	473.34	264.57	80242.95	1.00	0.63	177.52	34.93	38.63	76.89	0.62	0.00	10.94	1.53	51964.89
Range	6120.51	1.00	0.00	0.23	-0.19	-0.50	-0.43	-0.13	0.21	-0.20	-0.20	-0.20	1.00	1.00	-0.02	-0.01	-0.33
Payload	7473.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Liftoff distance	473.34	0.23	0.00	1.00	0.64	0.47	-0.33	0.49	-0.77	0.78	0.78	0.78	0.26	0.26	-0.19	-0.15	0.31
Landing distance	264.57	-0.19	0.00	0.64	1.00	0.34	-0.05	0.58	-0.57	0.92	0.92	0.92	-0.18	-0.18	-0.10	-0.08	0.27
Takeoff weight	80242.95	-0.50	0.00	0.47	0.34	1.00	-0.27	0.24	-0.42	0.38	0.38	0.38	-0.47	-0.47	0.00	0.00	0.81
Required weight quotient	1.00	-0.43	0.00	-0.33	-0.05	-0.27	1.00	-0.06	0.14	-0.09	-0.09	-0.09	-0.40	-0.40	0.02	0.01	-0.68
Buffeting speed quotient	0.63	-0.13	0.00	0.49	0.58	0.24	-0.06	1.00	-0.52	0.63	0.63	0.63	-0.12	-0.12	-0.13	-0.10	0.20
Optimum cruise speed	177.52	0.21	0.00	-0.77	-0.57	-0.42	0.14	-0.52	1.00	-0.83	-0.83	-0.83	0.20	0.20	0.15	0.14	-0.34
Landing speed	34.93	-0.20	0.00	0.78	0.92	0.38	-0.09	0.63	-0.83	1.00	1.00	1.00	-0.19	-0.19	-0.21	-0.17	0.31
Liftoff speed	38.63	-0.20	0.00	0.78	0.92	0.38	-0.09	0.63	-0.83	1.00	1.00	1.00	-0.19	-0.19	-0.21	-0.17	0.31
Stall speed	76.89	-0.20	0.00	0.78	0.92	0.38	-0.09	0.63	-0.83	1.00	1.00	1.00	-0.19	-0.19	-0.21	-0.17	0.31
Emissions	0.62	1.00	0.00	0.26	-0.18	-0.47	-0.40	-0.12	0.20	-0.19	-0.19	-0.19	1.00	1.00	-0.02	-0.01	-0.34
Consumption	0.00	1.00	0.00	0.26	-0.18	-0.47	-0.40	-0.12	0.20	-0.19	-0.19	-0.19	1.00	1.00	-0.02	-0.01	-0.34
Rotation	10.94	-0.02	0.00	-0.19	-0.10	0.00	0.02	-0.13	0.15	-0.21	-0.21	-0.21	-0.02	-0.02	1.00	0.46	0.00
Stability	1.53	-0.01	0.00	-0.15	-0.08	0.00	0.01	-0.10	0.14	-0.17	-0.17	-0.17	-0.01	-0.01	0.46	1.00	0.00
Cost	51964.89	-0.33	0.00	0.31	0.27	0.81	-0.68	0.20	-0.34	0.31	0.31	0.31	-0.34	-0.34	0.00	0.00	1.00

SYSTEM DETERMINANT

In Shannon 1948 the information channel it is described in the same way as the design relation, that is:

$$y = A \times x$$

The total information in y can then according to Shannon be calculated as:

$$H_y = -\log_2 \det A + H_x$$

Assuming a rectangular probability distribution and normalized design variables as inputs. This can be written as:

$$I_y = \det A_0 + I_x$$

or

$$I_y = I_A + I_x$$

SYSTEM DETERMINANT UNDER ROTATION

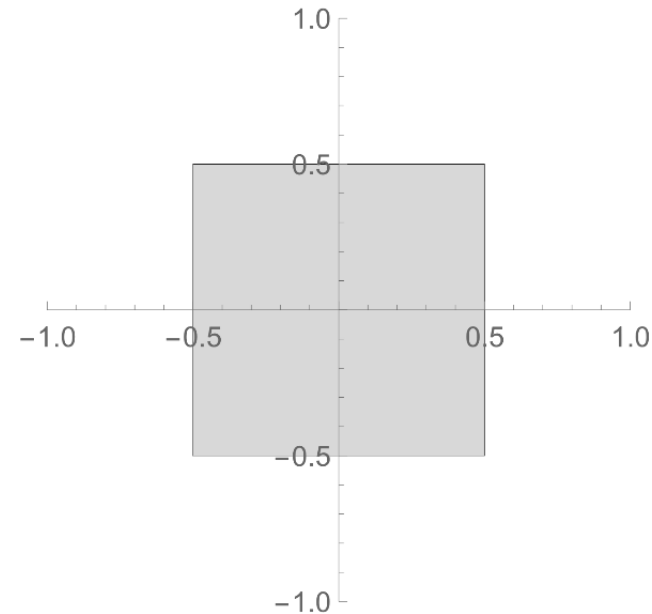
Consider the following system matrix:

$$\mathbf{A}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The determinant: $\det \mathbf{A}_0 = 1$

The correlation matrix is:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



SYSTEM DETERMINANT UNDER ROTATION

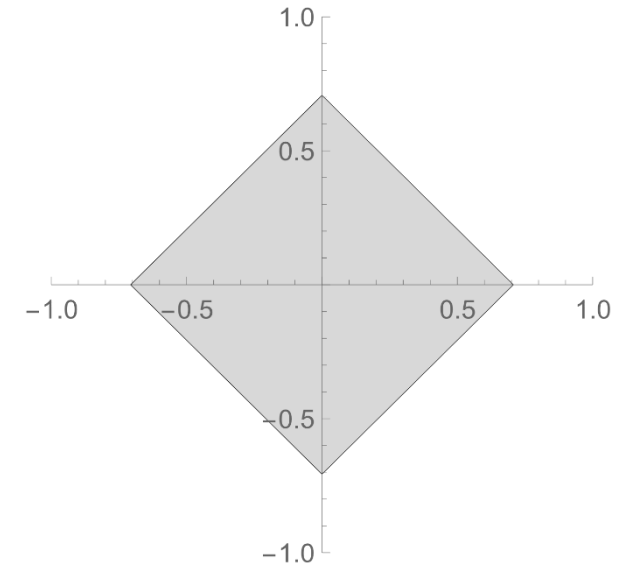
If the system matrix is rotated 45deg ($\pi/4$) it becomes

$$\mathbf{A}_0 = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

This is a fully coupled system. However, the determinant is still: $\det \mathbf{A}_0 = 1$

The correlation matrix is also invariant, and is for both cases:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



SYSTEM DETERMINANT: ALTERNATIVE EXEMPEL

(CHANGING SIGN OF $A_{0,12}$)

$$A_0 = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} - \epsilon \\ 1/\sqrt{2} - \epsilon & 1/\sqrt{2} \end{pmatrix}$$

If $\epsilon = 0$ the system determinant $\det A_0 = 0$.

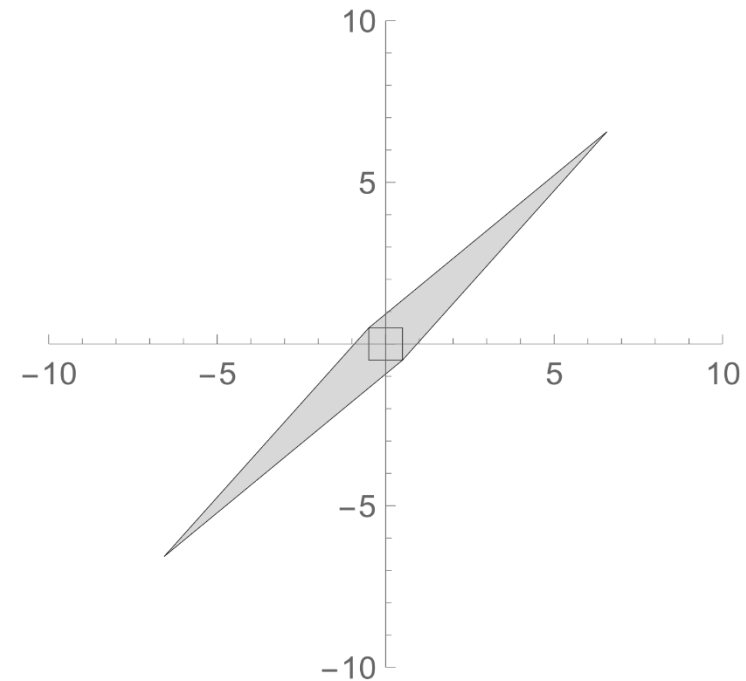
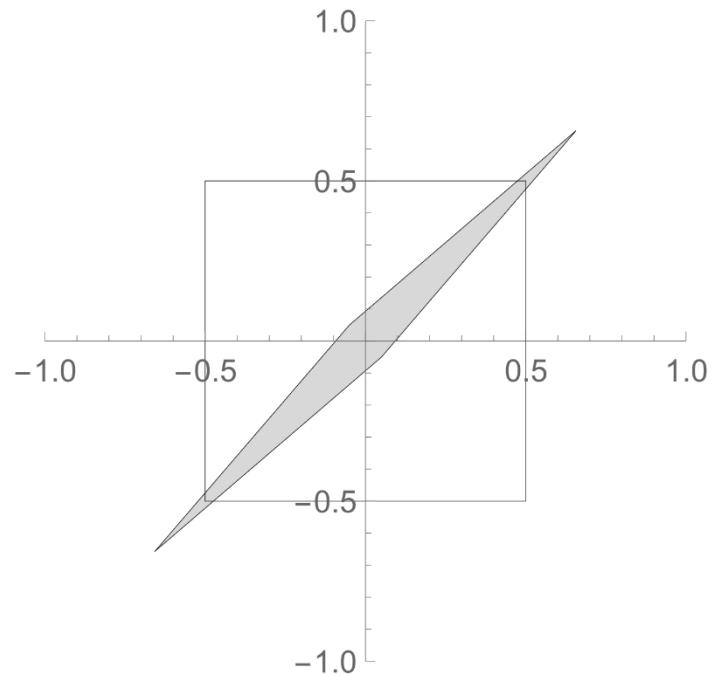
With $\epsilon = 0.1$ the system determinant is $\det A_0 = 0.131$

$$C = \begin{pmatrix} 1 & 0.988 \\ 0.988 & 1 \end{pmatrix}$$

The size of the projection in the functional space

$$S_{Fx} = \det A S_x$$

DESIGNSPACE FOR ALTERNATIVE EXAMPLE



Left: Projection in functional space of a design space of unit dimensions of a coupled design with high correlation. Right: Design space is increased to include the whole requirement range (of unit dimensions).

CONCLUSIONS

An uncoupled system the correlation matrix only has zero off-diagonal elements. However, there are also coupled systems that could be made uncoupled by rotating the coordinate system for the design parameters.

In this paper the functional correlation matrix and the system determinant of the design matrix has been shown to provide a deeper insights about the coupling of a system.

Furthermore, it is shown that using information theory there is a strong relationship between the two axioms in axiomatic design. I.e. an uncoupled system will have a low amount of wasted design space and require less design information compared to coupled ones. I.e. the shape of the requirement range does not fit to the design space.

DISCUSSION

A foundation for the argument of decoupling is that the functional characteristics are uncorrelated.

However, in design there are certainly a great deal of correlation between functional requirements.

E.g. in a product family there might be several product variants of different sizes, each with their functional characteristics that are more or less correlated to the size.

E.g. transport aircraft that are designed for a high passenger capacity also tend to be designed for a long range, indicating a correlation between these requirements.