Equation based model for losses in electric machines

A compact algebraic model for electric machine losses for applications in vehicle simulation and optimal control of vehicle speed

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Outline

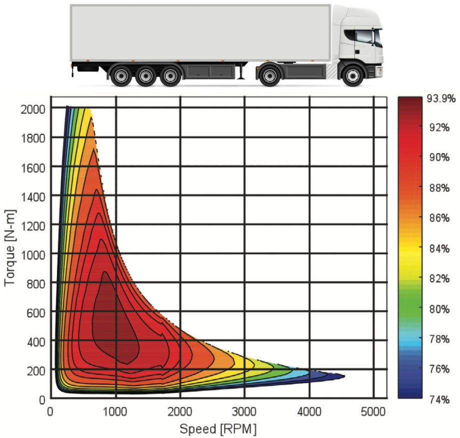
Context – Optimization and Simulation

The purpose and use

- Energy optimal control of vehicle propulsion
- Describe and adhere to machine constraints
- Software have algorithmic differentiation
- Utilize algebraic models for efficient optimization
- Energy consumption models that are algebraic

The problem and goal

- Detailed motor data not always available
- Efficiency (or loss) maps are often available
- Only speed and torque information available
- Develop an algebraic efficiency model suitable for efficient numerical optimal control



From data sheet: HVH410-15

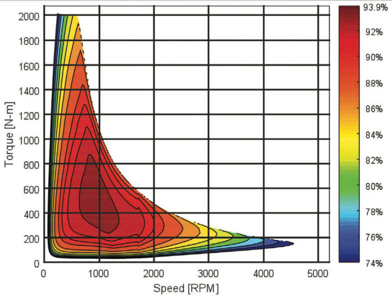
Outline

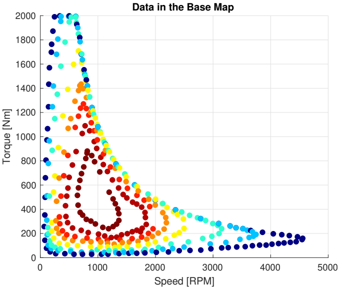
Development Procedure

- Model the machine losses
- Digitized efficiency lines of an available map –Data and points in the data digitized
- Study the behavior of the losses
- Identify behavior of losses
- Describe each term with equations
- Compile the model
- Develop fitting process

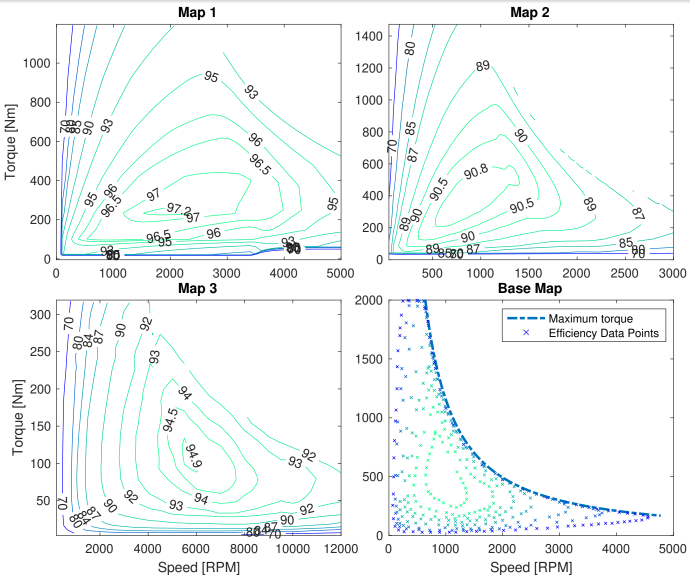
Goal – A model structure that generalizes

- Validate model structure and process on other maps containing new data





Four Example Maps - One Base Map



Four maps

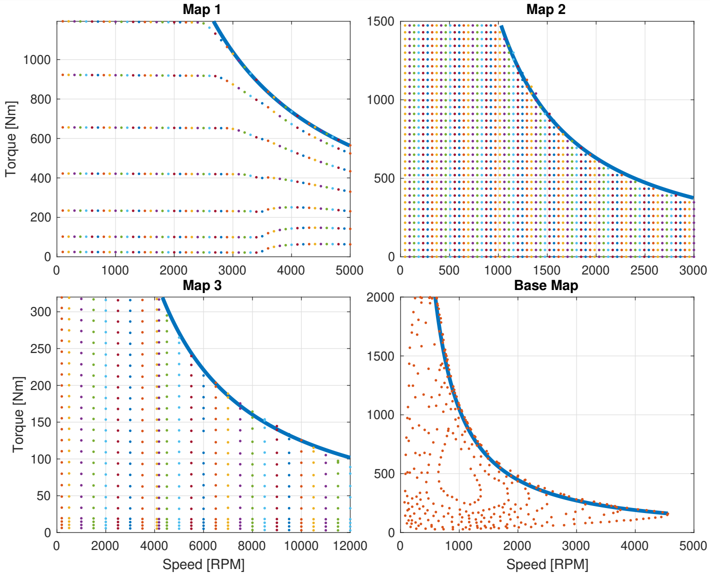
Varying shapes of iso-efficiency lines

- East–West
- South West–North East
- South East–North West
- North–South

–Can the model capture these?

Outline

Machine Data – Compute Losses from the Efficiency Maps

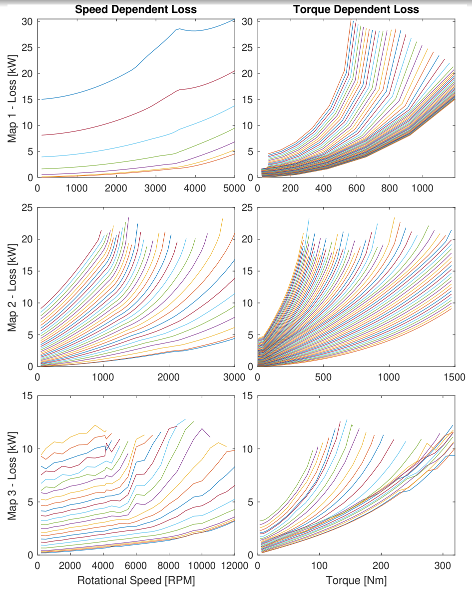


- Data from four machines
- Compute power losses
- $T, \omega, \eta, \Rightarrow \; P_l$

$$P_l = T\,\omega\,(\frac{1}{\eta} - 1)$$

- Maximum torque curve
- Regular and irregular grids

Power Losses – Torque and speed dependence



Trends for power losses

- Speed – Convex growth

$$P_{loss} = c_1\,\omega^2$$

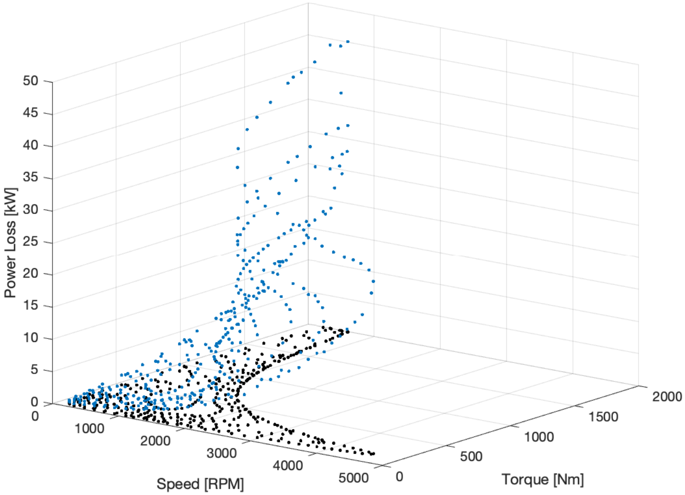
- Torque – Convex growth

$$P_{loss} = c_2\,T^2$$

- Losses are not shifted with offset, fan out at higher powers

$$P_{loss} = c_3\,\omega\,T$$

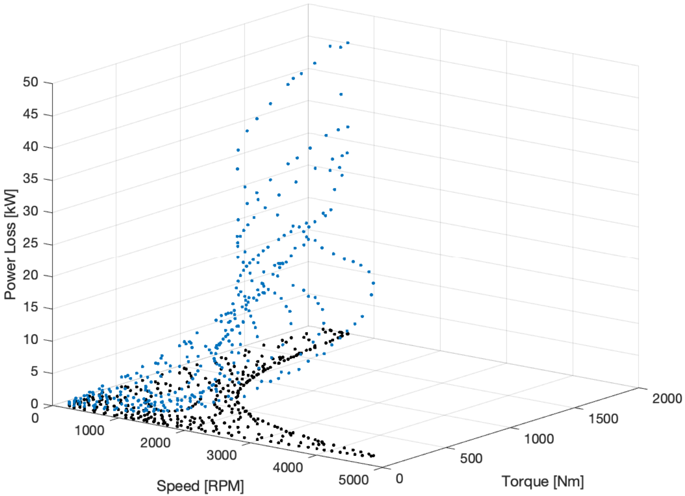
Observation – Accelerating Power Losses near Torque Max

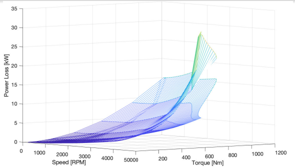


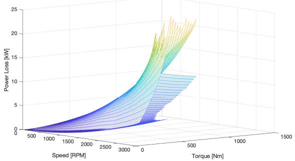
Power losses

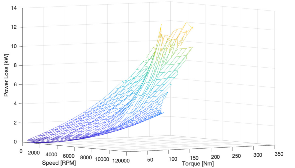
- Losses accelerate near max torque line
- Field weakening control

Confirming Accelerating Power Losses near Torque Max

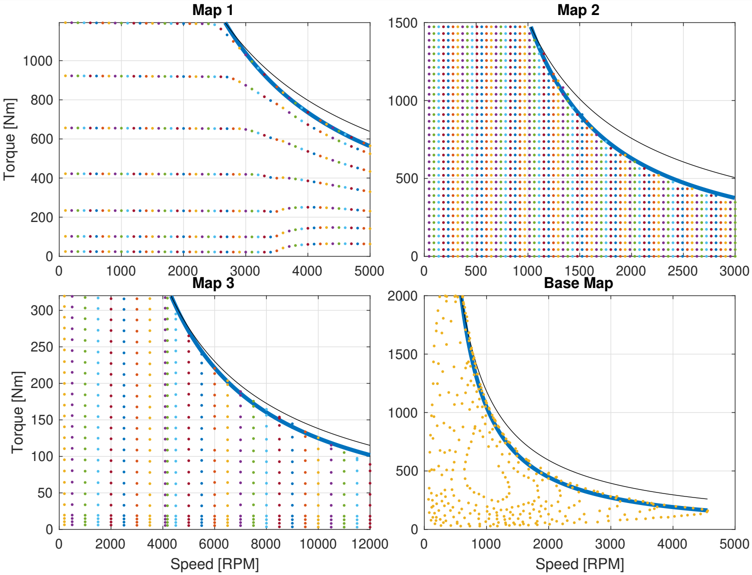








Maximum Torque Model



Max Torque Line

- Black - Max Power $T(\omega) = \frac{k}{\omega}$

Key result

- Blue - Power function $T(\omega) = \frac{k}{\omega^\alpha}$ $\alpha \approx 1.2$

Maximum Torque Function

The maximum torque, denoted $\widehat{T}$ is speed dependent and switches between two characteristics:

1. The maximum torque T_m up to design speed ω_m.
2. A speed dependent fall-off rate, following a power function in speed.

The maximum torque function can be expressed as

$$\widehat{T}(\omega) = \begin{cases} T_m, & \text{for } \omega \leq \omega_m \\ T_m \left(\frac{\omega_m}{\omega}\right)^{\alpha_T}, & \text{for } \omega > \omega_m \end{cases} \quad (1)$$

where the exponent can be determined from the maps and for the Base Map it is $\alpha_e \approx 1.226$.

The model has three parameters that can be determined from data. T_m is determined directly from the maximum torque in the map data or data sheet, while α_T and ω_m are tuned to data. A first estimate for ω_m is obtained from the maximum power P_m extracted either from the data sheet or map data, as $\omega_m = \frac{P_m}{T_m}$.

A first estimate for α_T is around 1.2, and ω_m is the design speed, but both ω_m and α_T might benefit from being adjusted to fit the data for best agreement.

The base expression for the maximum torque is expressed as

$$\widehat{T}(\omega) = \begin{cases} T_m, & \text{for } \omega \le \omega_m \\ T_m \left(\frac{\omega_m}{\omega}\right)^{\alpha_T}, & \text{for } \omega > \omega_m \end{cases}$$

The connection between T and ω at the maximum torque line can be expressed as.

$$T = T_m \left(\frac{\omega_m}{\omega}\right)^{\alpha_T}$$

This can be expressed as an implicit equation

$$\frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{\alpha_T} = 1 \quad \Leftrightarrow \quad \frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{\alpha_T} - 1 = 0$$

An accelerating loss can now be expressed using this function in an exponential.

$$P_l = e^{a_5\left(\frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{a_6} - a_7\right)} = e^{-a_5\, a_7}\, e^{a_5\, \frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{a_6}} = c_4\, e^{c_5\, \frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{c_6}}$$

where c_5, c_6, and α_T are parameters used to fine-tune the shape of the growth.

Outline

The Resulting Model Structure

The complete model, based on the observations, is expressed as

$$P_l(\omega, T) = c_0 + c_1\,\omega^2 + c_2\,T^2 + c_3\,\omega\,T + c_4\,e^{c_5\,\frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{c_6}}$$

Seven (7) tuning parameters where 5 appear linearly.

- Solving full equation with nonlinear least squares had problems to converge.
- Convergence depended highly on initial conditions.
- Separate into two problems, nonlinear outer ($i \in [5, 6]$) and linear inner problem.

Outline

Model Tuning Algorithm

The complete model equation

$$P_l(\omega, T) = c_0 + c_1\,\omega^2 + c_2\,T^2 + c_3\,\omega\,T + c_4\,e^{c_5\,\frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{c_6}} \tag{2}$$

5 parameters appear linearly $c_i,\ i \in [0-4]$, 3 parameter appear nonlinearly $c_i,\ i \in [5-7]$.

- Give initial guess on the three parameters $c_i,\ i \in [5-6]$.
- Iterate NLLS with Marquart-Levenberg (ML) algorithm for 3 parameters $c_i,\ i \in [5-7]$.
- Internally in the loss function used by ML when $c_i,\ i \in [5-6]$ are given, the parameters $c_i,\ i \in [0-4]$ can be determined with the linear least squares method.

Future Work

- Compare fitting in Power domain and η domain
- Evaluate the model and method on more maps

Outline

Summary of Model components

Maximum Torque Function (constraints in optimal control)

$$\hat{T}(\omega) = \begin{cases} T_m, & \text{for } \omega \leq \omega_m \\ T_m \left(\frac{\omega_m}{\omega}\right)^{\alpha_T}, & \text{for } \omega > \omega_m \end{cases}$$

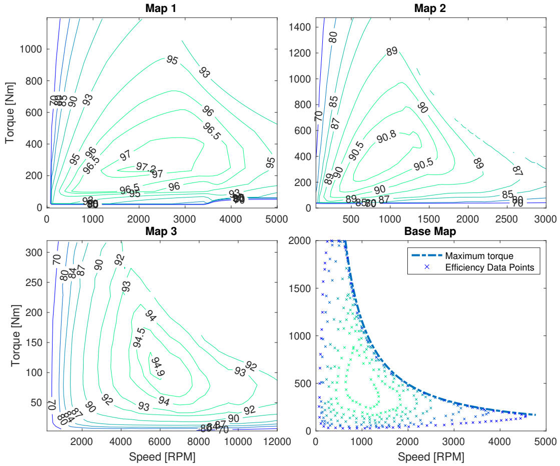
Two parameters to determine ω_m and α_T

Power Loss Model (Fuel Economy)

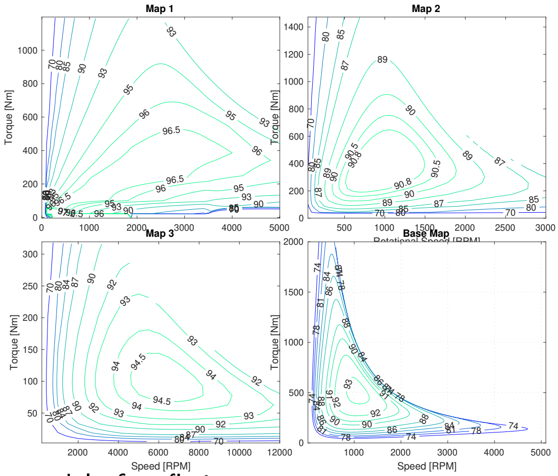
$$P_l(\omega, T) = c_0 + c_1\,\omega^2 + c_2\,T^2 + c_3\,\omega\,T + c_4\,e^{c_5 \frac{T}{T_m}\left(\frac{\omega}{\omega_m}\right)^{c_6}}$$

Seven unknown parameters to determine $c_i,\ i \in [0-6]$

Original Maps



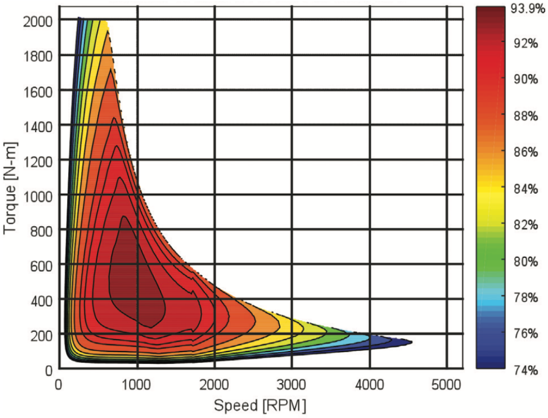
Models of the Maps



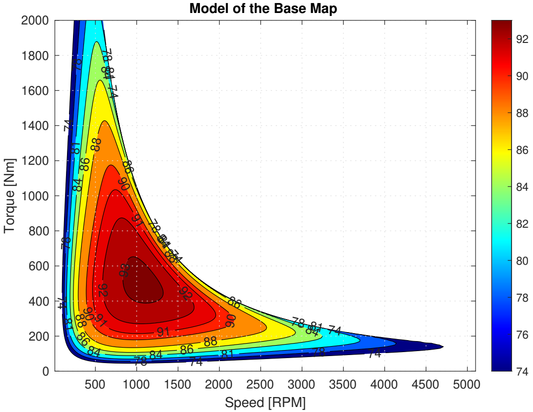
Visual impresson of the maps are preserved in the model after fitting.

Comparison

Original Base Map



Model of the Base Map



Outline

Conclusions

Current progress

- A compact and analytic model for electric machine power consumption has been developed
- The model describes the machine losses using base functions
  - Maximum torque function – Power function needed
  - Machine loss model based on nonlinear regression vectors and an implicit max torque function in an exponential
- A Marquardt-Levenberg based parameter-tuning method developed
- Visualization of the resulting model shows good qualitative agreement with the efficiency maps

Future work

- Compare fitting in Power domain and η domain
- Test robustness of the method and cover more machine types and efficiency maps
- Evaluate model flexibility and parameter variations

The End – Questions and discussion

Questions?