Transmission Line Modelling,TLM, for System Simulation

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Some Industrial Partners and Applications

Aircaft

Saab AB

Construction Machines

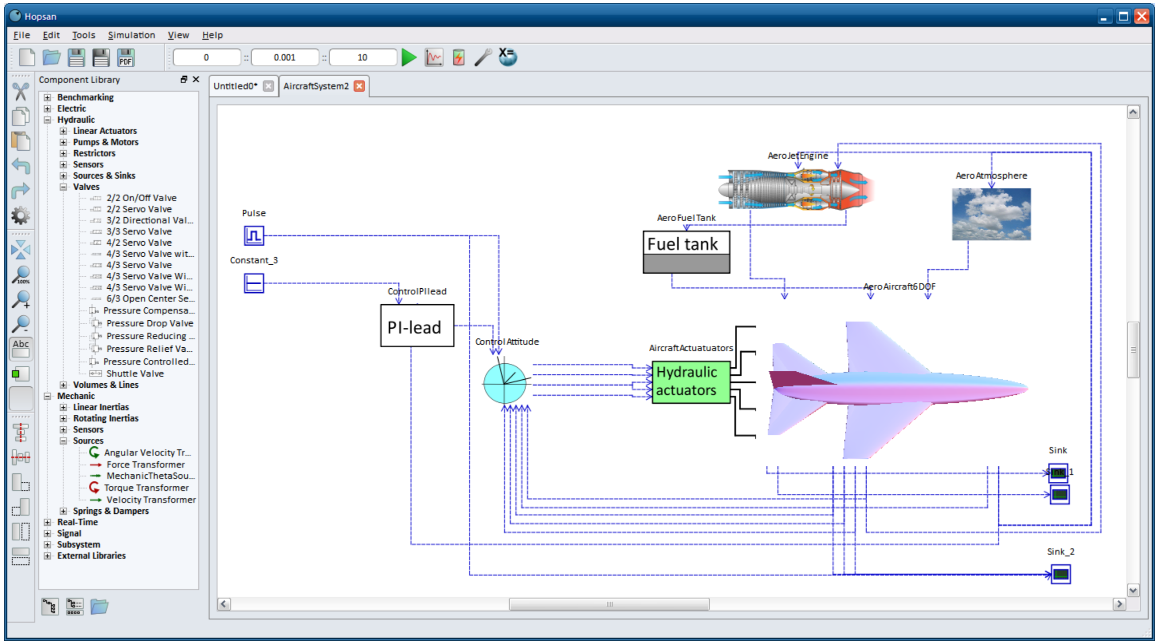
Volvo CE

Rock drills

Atlas Copco

Multi domain dynamic system simulation

- Oil hydraulics
- Gas, pneumatics
- Thermal
- Electrical power
- Mechanical
- Flight dynamics
- Control systems
- Propulsion



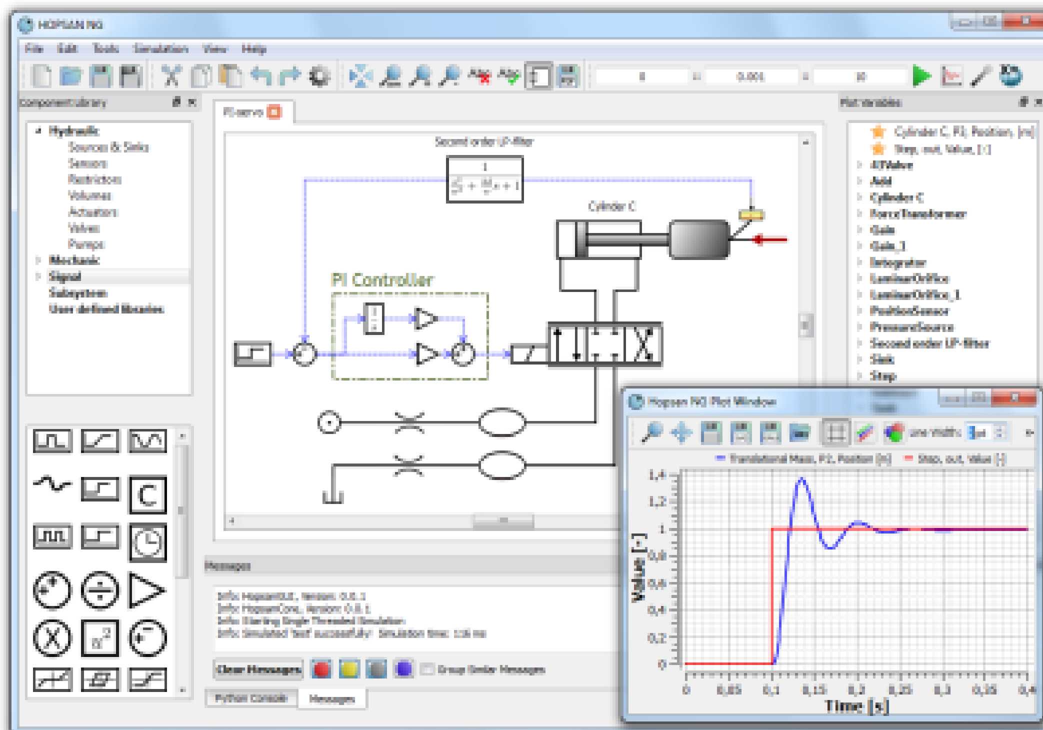
- Real systems tend to become very complex and multidisciplinary
- Many systems (e.g. hydraulics) are prone to be numerically stiffness, strong nonlinearites, and discontinuities.

Hopsan Development

- Software for system simulation. Hydraulic, mechanical, electrical, control systems, thermal, etc.
- Work on first Hopsan (in Fortran) began in late 1970s at Linköping University
- Used by industry and for research
- Development of new version called Hopsan NG (in C++) began in 2009
- Longest running simulation software with continous development *in the world* (?)
- Current version: Beta 0.7
- Open source that can be downloaded from http://www.iei.liu.se/flumes/system-simulation/hopsanng

Hopsan NG

- Genuine team work at Flumes
- Freeware that can be downloaded from http://www.iei.liu.se/flumes/system-simulation/hopsanng



Bilateral Delay Lines or *Transmission Line Modelling* (TLM)

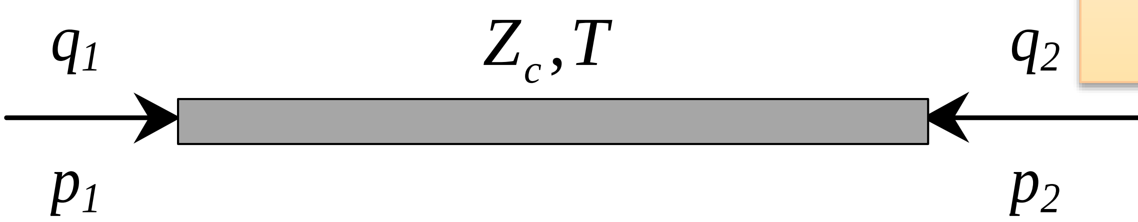
- D M Auslander, 'Distributed System Simulation with Bilateral Delay-Line Models' Journal of Basic Engineering, Trans. ASME p195-p200, June 1968.
- P. B. Johns and M.O'Brien. 'Use of the transmission line modelling (t.l.m) method to solve nonlinear lumped networks.' *The Radio Electron and Engineer.* 1980.
- P Krus, A Jansson, J-O Palmberg, K Weddfeldt. 'Distributed Simulation of Hydromechanical Systems'. Presented at 'Third Bath International Fluid Power Workshop', Bath, UK 1990.
- Burton, J. D., Edge, K. A., Burrows, C. R., 1993. Partitioned simulation of hydraulic systems using transmission-line modelling. *In*: *ASME WAM*, New Orleans, LA, USA. Publ by ASME, New York, NY, USA*. (Using Transputer technology*)

The transmission line

Can represent:

Electrical line, hydraulic line, pneumatic line, spring.

Both capacitance and inductance



$$p_1(t) = p_2(t-T) + Z_c\left(q_1(t) + q_2(t-T)\right)$$

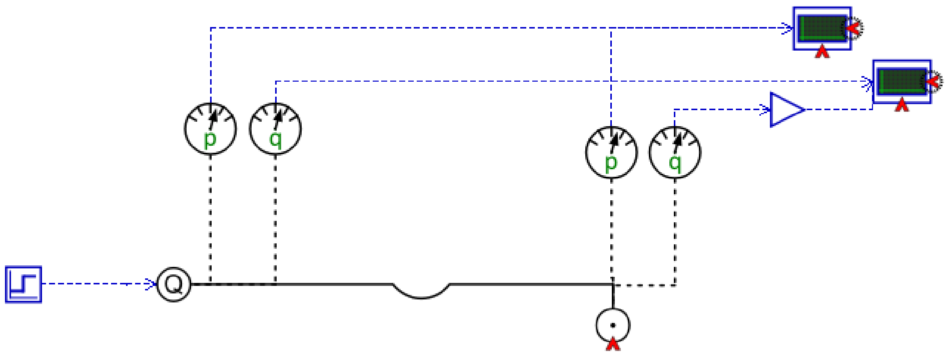
$$p_2(t) = p_1(t-T) + Z_c\left(q_2(t) + q_1(t-T)\right)$$

Alternatively

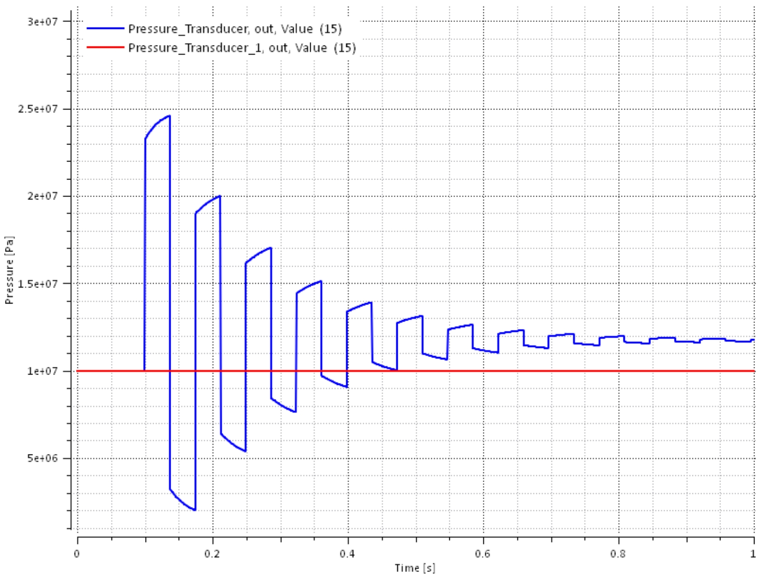
$$p_1(t) = c_1(t) + Z_c q_1(t) \qquad c_1(t) = p_2(t-T) + Z_c q_2(t-T)$$

$$p_2(t) = c_2(t) + Z_c q_2(t) \qquad c_2(t) = p_1(t-T) + Z_c q_1(t-T)$$

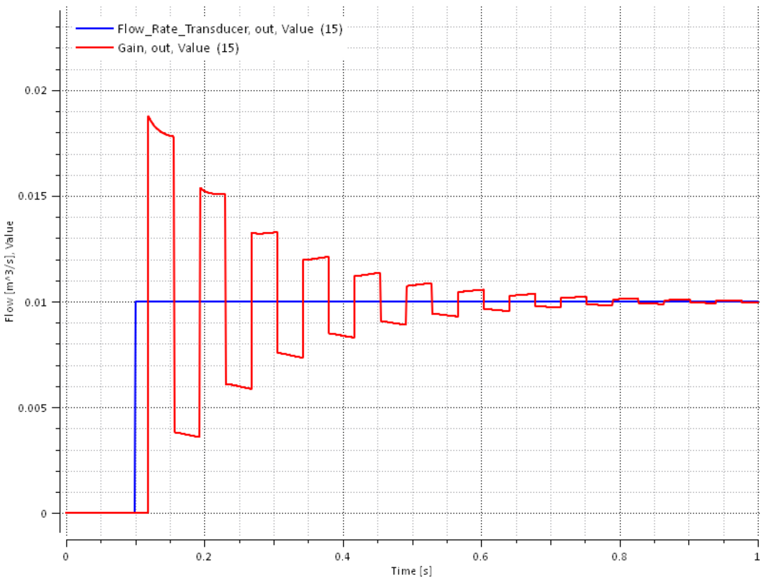
Simulation of long transmission line



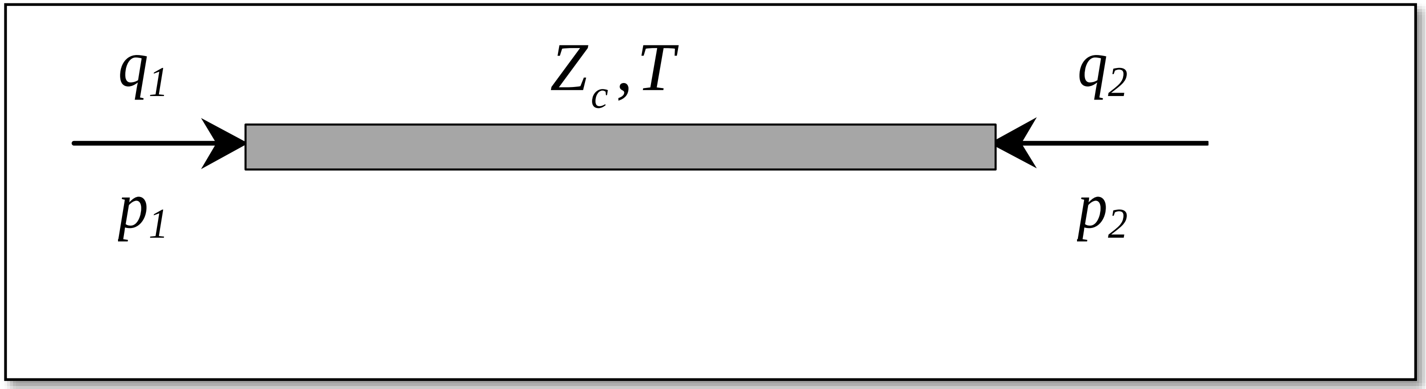
Pressure



Flows



The transmission line



Capacitance and inductance as a function of chacteristic impedance and delay time

$$C = \frac{T}{Z_c}$$

$$L = Z_c T$$

$$Z_c = \sqrt{\frac{L}{C}}$$

$$T = \sqrt{LC}$$

If time step and capacitance are known

$$Z_c = \frac{T}{C}$$

$$L_p = \frac{T^2}{C}$$

Parasitic inductance

If time step and inductance are known

$$Z_c = \frac{L}{T}$$

$$C_p = \frac{T^2}{L}$$

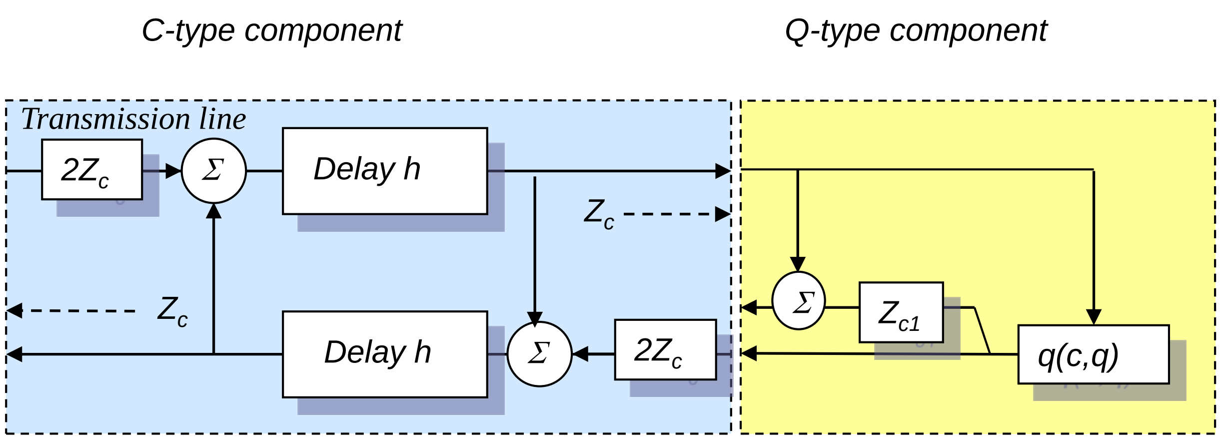
Parasitic capacitance

Block Diagram

$$c_1(t) = p_2(t-T) + Z_c q_2(t-T)$$
$$c_2(t) = p_1(t-T) + Z_c q_1(t-T)$$

$$p_1(t) = c_1(t) + Z_c q_1(t)$$
$$p_2(t) = c_2(t) + Z_c q_2(t)$$

$$c_1(t) = c_2(t-T) + 2Z_c q_2(t-T)$$
$$c_2(t) = c_1(t-T) + 2Z_c q_1(t-T)$$



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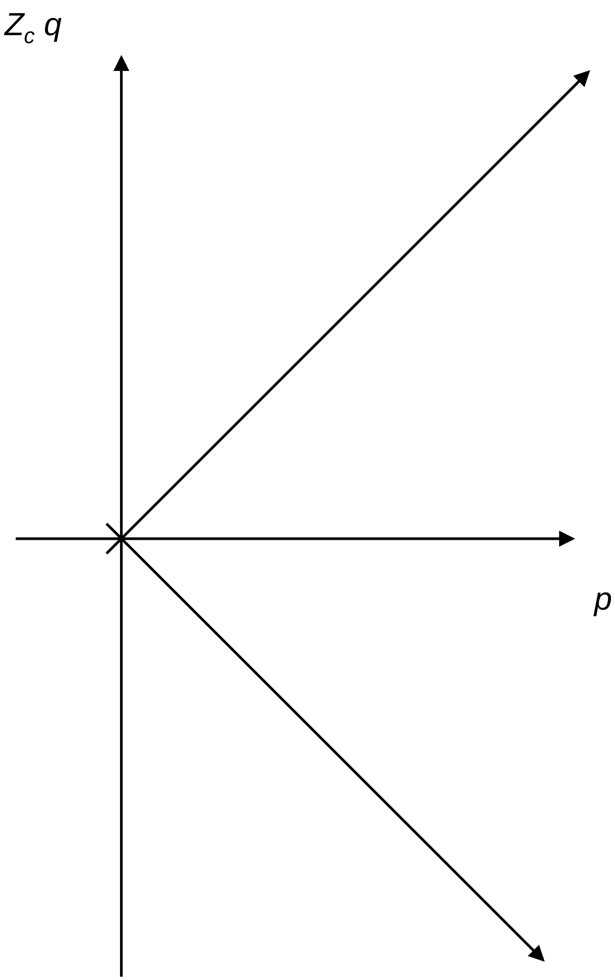
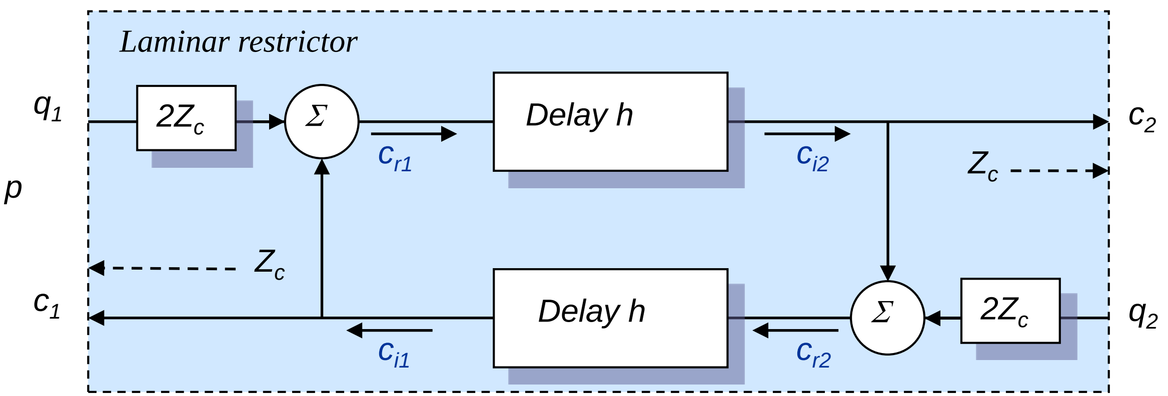
Relations between p, q and ci, ,cr



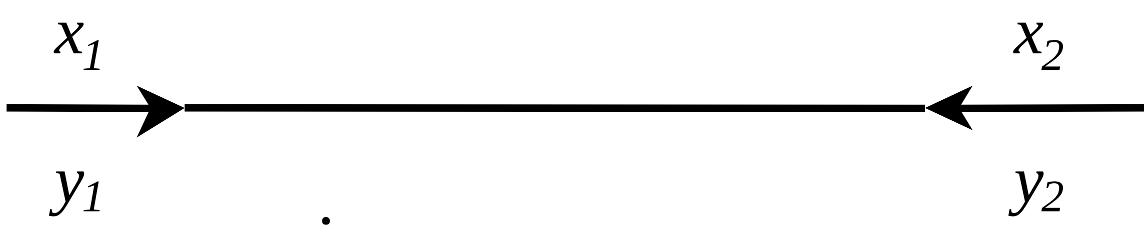
$$c_r = p + Z_c q$$

$$(c_r = \frac{1}{\sqrt{Z_c}} p + \sqrt{Z_c} q)$$



$$c_i = p - Z_c q$$

The Transmission Line as a General Integrator

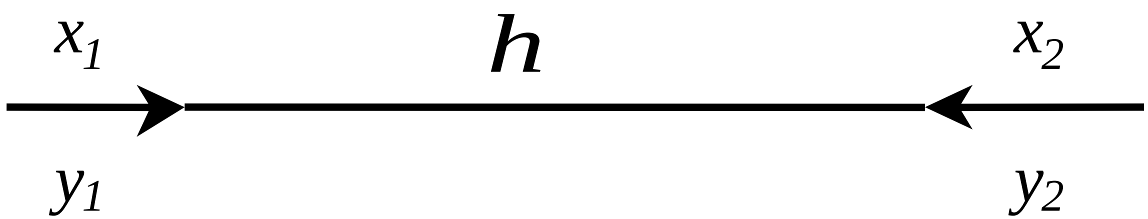


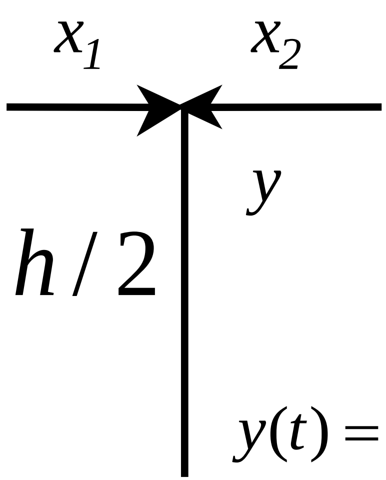
$$\dot{y} = x_1 + x_2$$

$$y_1(t) = y_2(t-T) + h\left(x_1(t) + x_2(t-T)\right)$$

$$y_2(t) = y_1(t-T) + h\left(x_2(t) + x_1(t-T)\right)$$

The Trapezoidal Rule as an Integrator



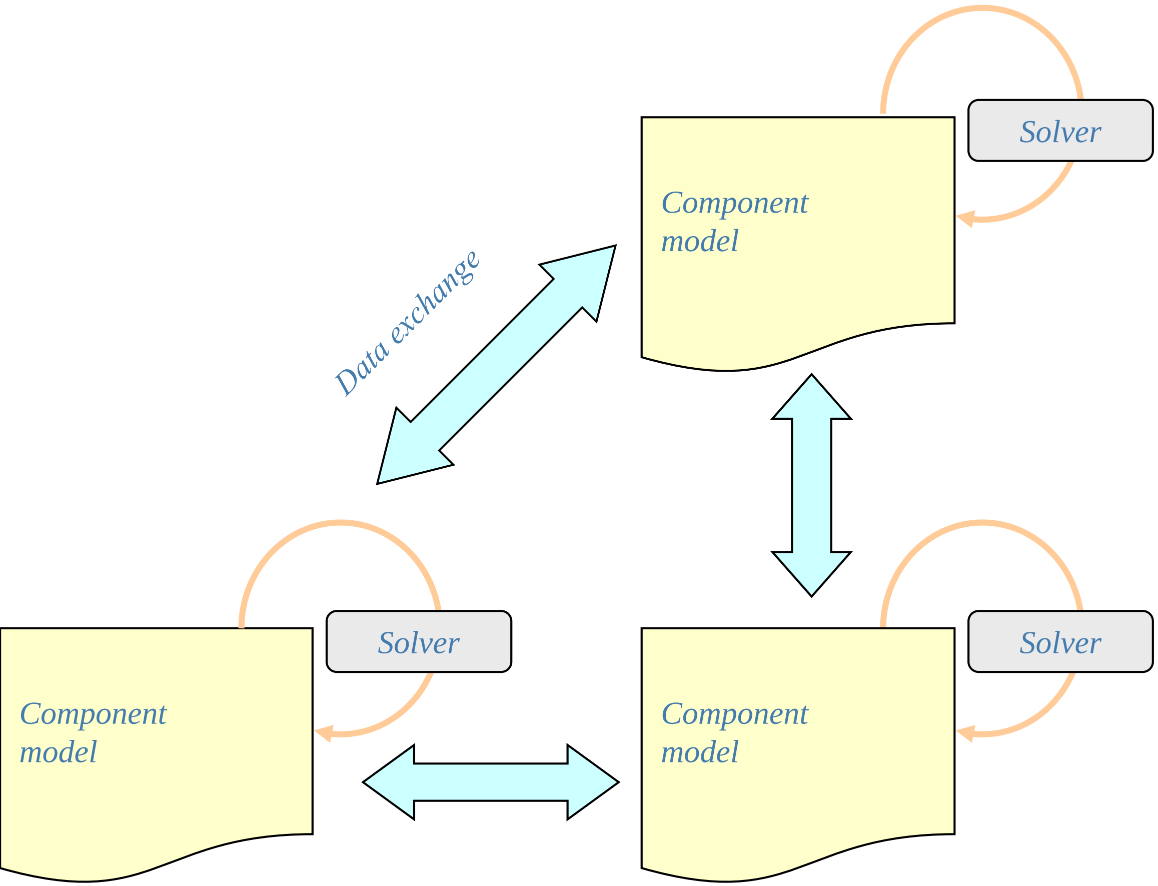


$$h' = h/2$$

$$\dot{y} = x_1 + x_2$$

$$y(t) = y(t-h) + h(x_1(t) + x_2(t) + x_1(t-h) + x_2(t-h))$$

Distributed Model Structure



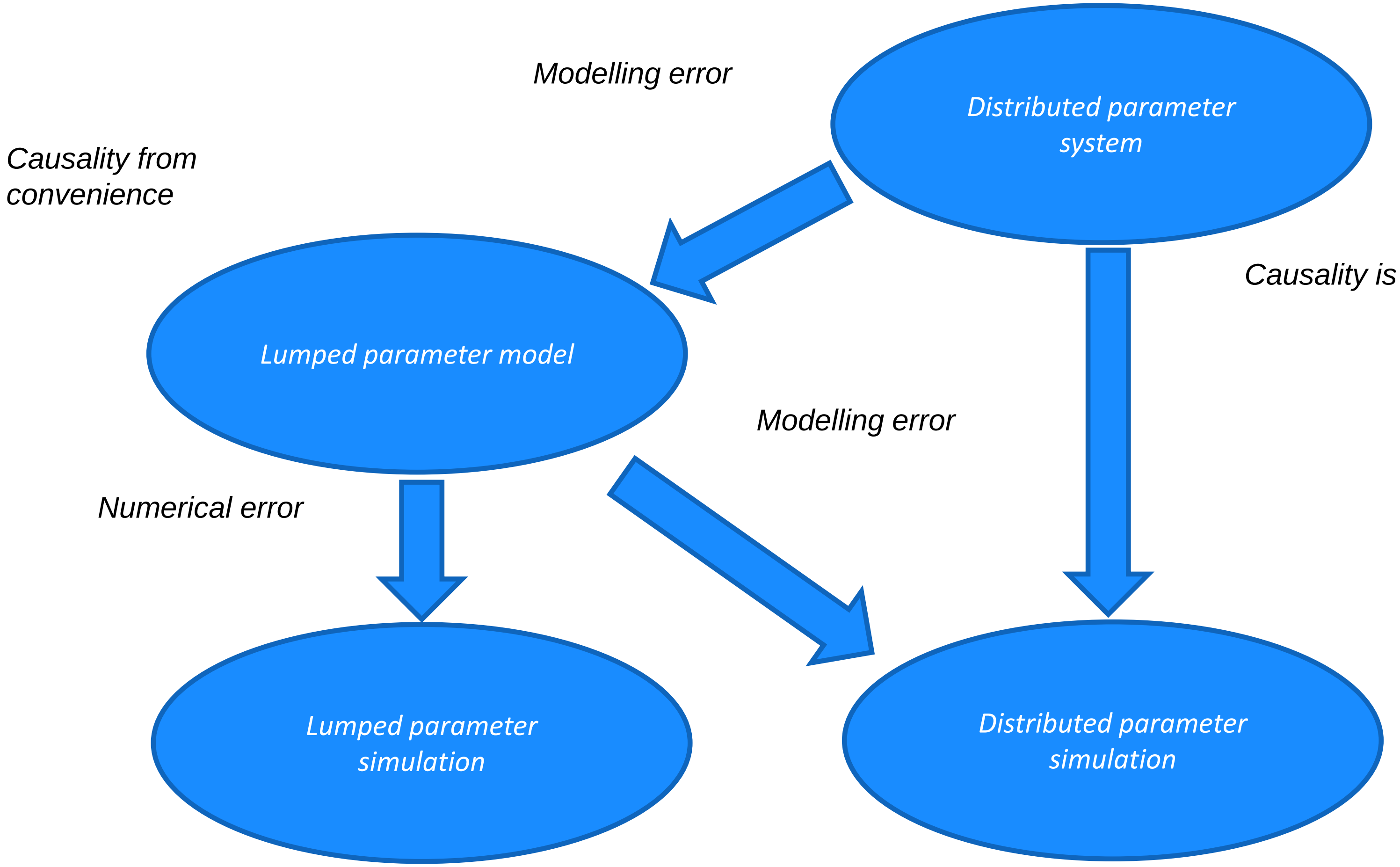
Individual simulation set-up

Ideal for using multi-core architectures

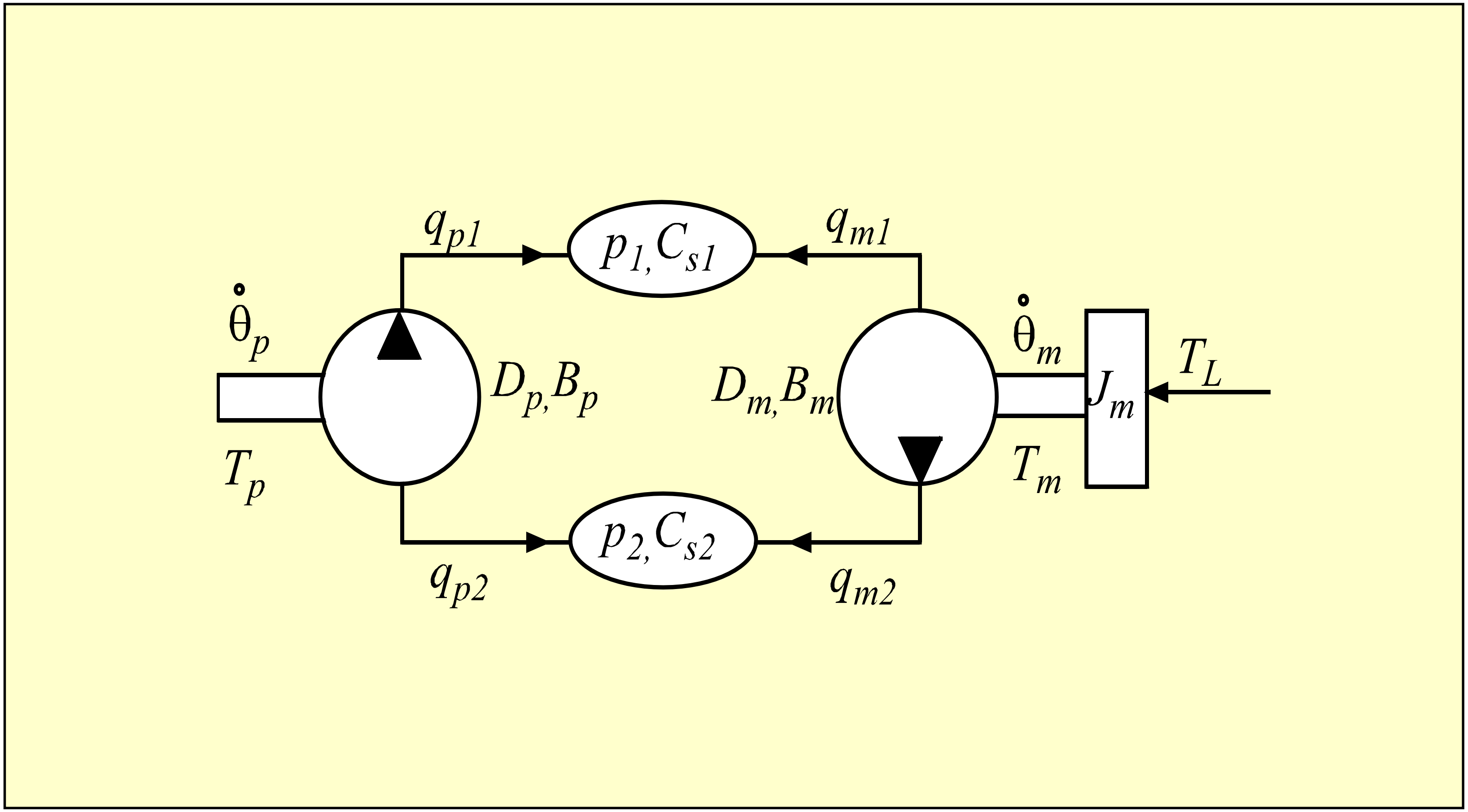
Enables interdisciplinary model development and simulation

Efficient for large systems

Relations between distributed and lumped parameter modelling



Example: Pump-motor system



F(y,dy/dt) and y for the transmission

$$F(y,\dot{y},u,t)=\begin{pmatrix} \dot{\theta}_p B_p + \dfrac{p_a - p_b}{D_p} - T_p \\ -\dot{\theta}_p D_p + q_{p1} \\ \dot{\theta}_p D_p + q_{p2} \\ \dot{p}_1 - \dfrac{q_{m1}+q_{p1}}{C_{s1}} \\ \dot{p}_2 - \dfrac{q_{m2}+q_{p2}}{C_{s2}} \\ -\dot{\theta}_m D_m + q_{m1} \\ \dot{\theta}_m D_m + q_{m2} \\ -\dot{\theta}_m B_m + \dfrac{p_a - p_b}{D_m} - T_m \\ \ddot{\theta}_m + \dfrac{B_m}{J_m}\dot{\theta}_m - \dfrac{p_a - p_b}{J_m D_m} + \dfrac{T_L}{J_m} \end{pmatrix}$$

$$y=\begin{pmatrix} T_p \\ q_{p1} \\ q_{p2} \\ p_1 \\ p_2 \\ q_{m1} \\ q_{m2} \\ T_m \\ \theta_m \end{pmatrix}$$

Differential algebraic equations

$$F(x, \dot{x}, u, t) = 0$$

$$F\left(y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, \ldots, \frac{d^2y}{dt^2}, t\right) = 0$$

$$\frac{d}{dt} = \frac{2\left(1 - q^{-1}\right)}{h\left(1 + q^{-1}\right)}$$

Bilinear transform (derived from the trapezoidal rule, second order Runge-Kutta)

$$qy = y(t + h)$$

q is the time displacement operator

Differential algebraic equations

$$F\left(y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, \ldots, \frac{d^2y}{dt^2}, t\right) = 0$$

can be transformed into:

$$G(y(t), y(t-h), \ldots, y(t-nh), u(t), u(t-h), \ldots, u(t-nh), t) = 0$$

$$G(y(t), t) = 0$$

Use Newton-Rapson

$$y_{k+1} = y_k(t) - J_k(t)^{-1} G(y_k(t))$$

$$J_{ijk} = \frac{\partial G_i(y_k(t))}{\partial y_j}$$

Compare: Differential algebraic equations with lumped parameters

$$G(y, t) =$$

$$\begin{pmatrix}
\mathrm{DS}[1, -h\, p_a + h\, p_b + D_p\,(h\, T_p + 2\, B_p\, \theta_p)] - h\, p_a + h\, p_b + h\, D_p\, T_p - 2\, B_p\, D_p\, \theta_p \\
\mathrm{DS}[1, h\, q_{\mathrm{p1}} + 2\, D_p\, \theta_p] + h\, q_{\mathrm{p1}} - 2\, D_p\, \theta_p \\
\mathrm{DS}[1, h\, q_{\mathrm{p2}} - 2\, D_p\, \theta_p] + h\, q_{\mathrm{p2}} + 2\, D_p\, \theta_p \\
\mathrm{DS}[1, -2\, C_{\mathrm{s1}}\, p_a - h\,(q_{\mathrm{m1}} + q_{\mathrm{p1}})] + 2\, C_{\mathrm{s1}}\, p_a - h\,(q_{\mathrm{m1}} + q_{\mathrm{p1}}) \\
\mathrm{DS}[1, -2\, C_{\mathrm{s2}}\, p_b + h\,(q_{\mathrm{m2}} + q_{\mathrm{p2}})] + 2\, C_{\mathrm{s2}}\, p_b + h\,(q_{\mathrm{m2}} + q_{\mathrm{p2}}) \\
\mathrm{DS}[1, h\, q_{\mathrm{m1}} - 2\, D_m\, \theta_m] + h\, q_{\mathrm{m1}} + 2\, D_m\, \theta_m \\
\mathrm{DS}[1, h\, q_{\mathrm{m2}} + 2\, D_m\, \theta_m] + h\, q_{\mathrm{m2}} - 2\, D_m\, \theta_m \\
\mathrm{DS}[1, -h\, p_a + h\, p_b + D_m\,(h\, T_m - 2\, B_m\, \theta_m)] - h\, p_a + h\, p_b + h\, D_m\, T_m + 2\, B_m\, D_m\, \theta_m \\
\mathrm{DS}[1, 2(-h^2\, p_a + h^2\, p_b + D_m\,(h^2\, T_L - 4\, J_m\, \theta_m))] + \mathrm{DS}[2, -h^2\, p_a + h^2\, p_b + D_m\,(h^2\, T_L - \\
2\, h\, B_m\, \theta_m + 4\, J_m\, \theta_m)] - h^2\, p_a + h^2\, p_b + h^2\, D_m\, T_L + 2\, h\, B_m\, D_m\, \theta_m + 4\, D_m\, J_m\, \theta_m
\end{pmatrix}$$

Jacobian of the lumped time discrete algebraic system

$$J = \begin{pmatrix} h\,D_p & 0 & 0 & -h & h & 0 & 0 & 0 & 0 \\ 0 & h & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -h & 0 & 2\,C_{s1} & 0 & -h & 0 & 0 & 0 \\ 0 & 0 & h & 0 & 2\,C_{s2} & 0 & h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h & 0 & 0 & 2\,D_m \\ 0 & 0 & 0 & 0 & 0 & 0 & h & 0 & -2\,D_m \\ 0 & 0 & 0 & -h & h & 0 & 0 & h\,D_m & 2\,B_m\,D_m \\ 0 & 0 & 0 & -h^2 & h^2 & 0 & 0 & 0 & 2\,h\,B_m\,D_m + 4\,D_m\,J_m \end{pmatrix}$$

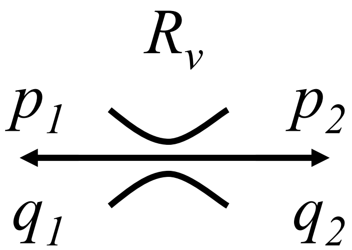
Time discrete algebraic system with distributed parameters

$$G(y,t)=\begin{pmatrix}
\mathrm{DS}[1,-h\,p_a+h\,p_b+D_p(h\,T_p+2\,B_p\,\theta_p)]-h\,p_a+h\,p_b+h\,D_p\,T_p-2\,B_p\,D_p\,\theta_p\\
\mathrm{DS}[1,h\,q_{\mathrm{p1}}+2\,D_p\,\theta_p]+h\,q_{\mathrm{p1}}-2\,D_p\,\theta_p\\
\mathrm{DS}[1,h\,q_{\mathrm{p2}}-2\,D_p\,\theta_p]+h\,q_{\mathrm{p2}}+2\,D_p\,\theta_p\\
-\mathrm{DS}[1,p_{\mathrm{a2}}]+p_{\mathrm{a1}}+(\mathrm{DS}[1,q_{\mathrm{m1}}]-q_{\mathrm{p1}})Z_c\\
-\mathrm{DS}[1,p_{\mathrm{b2}}]+p_{\mathrm{b1}}+(\mathrm{DS}[1,q_{\mathrm{m2}}]-q_{\mathrm{p2}})Z_c\\
-\mathrm{DS}[1,p_{\mathrm{a1}}]+p_{\mathrm{a2}}+(\mathrm{DS}[1,q_{\mathrm{p1}}]-q_{\mathrm{m1}})Z_c\\
-\mathrm{DS}[1,p_{\mathrm{b1}}]+p_{\mathrm{b2}}+(\mathrm{DS}[1,q_{\mathrm{p2}}]-q_{\mathrm{m2}})Z_c\\
\mathrm{DS}[1,h\,q_{\mathrm{m1}}-2\,D_m\,\theta_m]+h\,q_{\mathrm{m1}}+2\,D_m\,\theta_m\\
\mathrm{DS}[1,h\,q_{\mathrm{m2}}+2\,D_m\,\theta_m]+h\,q_{\mathrm{m2}}-2\,D_m\,\theta_m\\
\mathrm{DS}[1,-h\,p_a+h\,p_b+D_m(h\,T_m-2\,B_m\,\theta_m)]-h\,p_a+h\,p_b+h\,D_m\,T_m+2\,B_m\,D_m\,\theta_m\\
\mathrm{DS}[1,2(-h^2\,p_a+h^2\,p_b+D_m(h^2\,T_L-4\,J_m\,\theta_m))]+\mathrm{DS}[2,-h^2\,p_a+h^2\,p_b+\\
D_m(h^2\,T_L-2\,h\,B_m\,\theta_m+4\,J_m\,\theta_m)]-h^2\,p_a+h^2\,p_b+h^2\,D_m\,T_L+\\
2\,h\,B_m\,D_m\,\theta_m+4\,D_m\,J_m\,\theta_m
\end{pmatrix}$$

Jacobian of the distributed time discrete algebraic system

$$
J = \begin{pmatrix}
h\,D_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -Z_c & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -Z_c & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -Z_c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -Z_c & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & h & 0 & 0 & 2\,D_m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h & 0 & -2\,D_m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h\,D_m & 2\,B_m\,D_m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\,h\,B_m\,D_m + 4\,J_m\,D_m
\end{pmatrix}
$$

Laminar restrictor with transmission line boundaries



$$q_2 = \frac{p_1 - p_2}{R_v}$$

$$q_1 = -q_2$$

From transmission line

$$p_1 = c_1 + Z_c q_1$$

$$p_2 = c_2 + Z_c q_2$$

Solved for p1 and p2

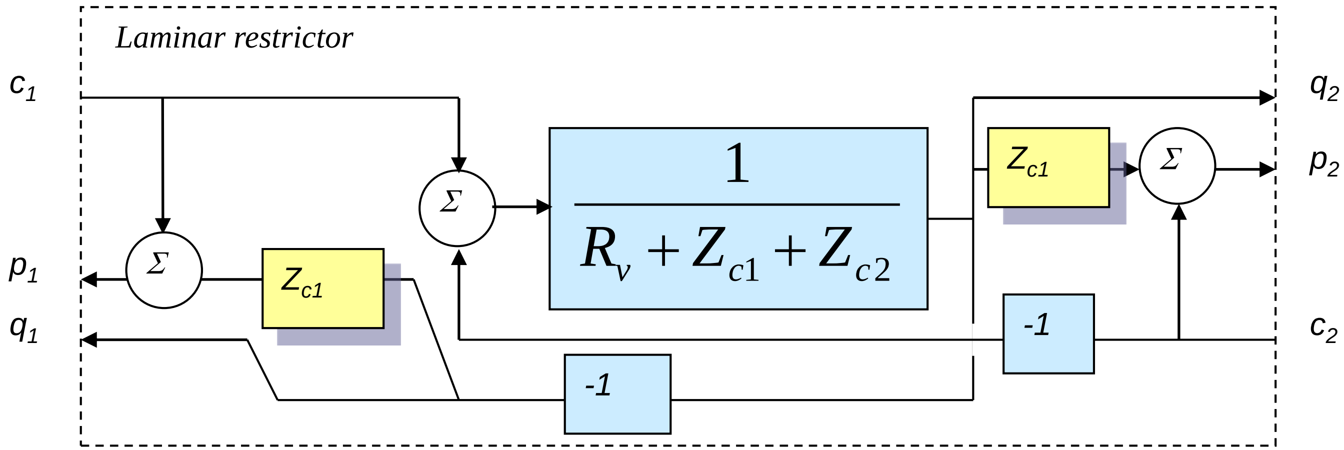
$$q_2 = \frac{c_1 - c_2}{R_v + Z_{c1} + Z_{c2}}$$

$$q_1 = -q_2$$

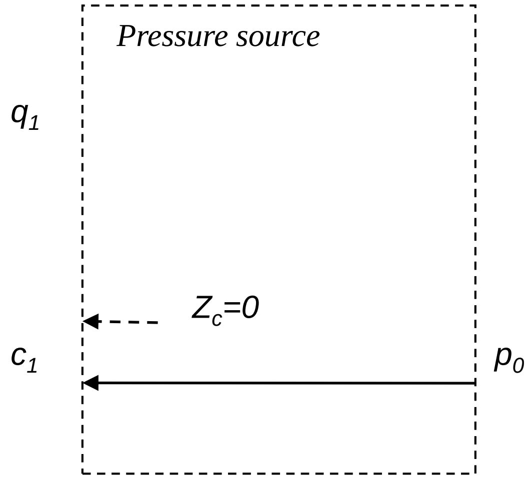
$$p_1 = c_1 + Z_c q_1$$

$$p_2 = c_2 + Z_c q_2$$

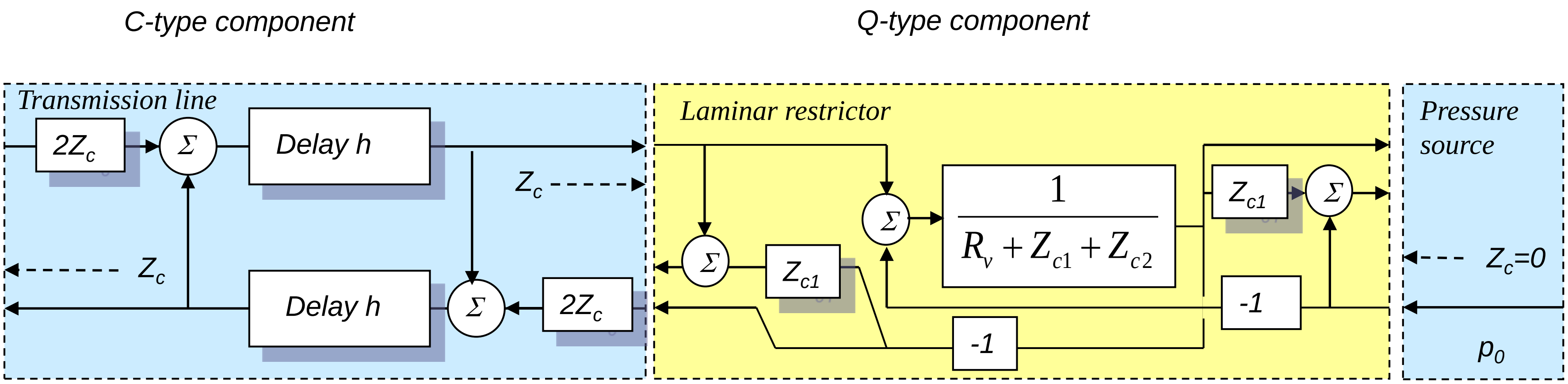
Blockdiagram of laminar restrictor



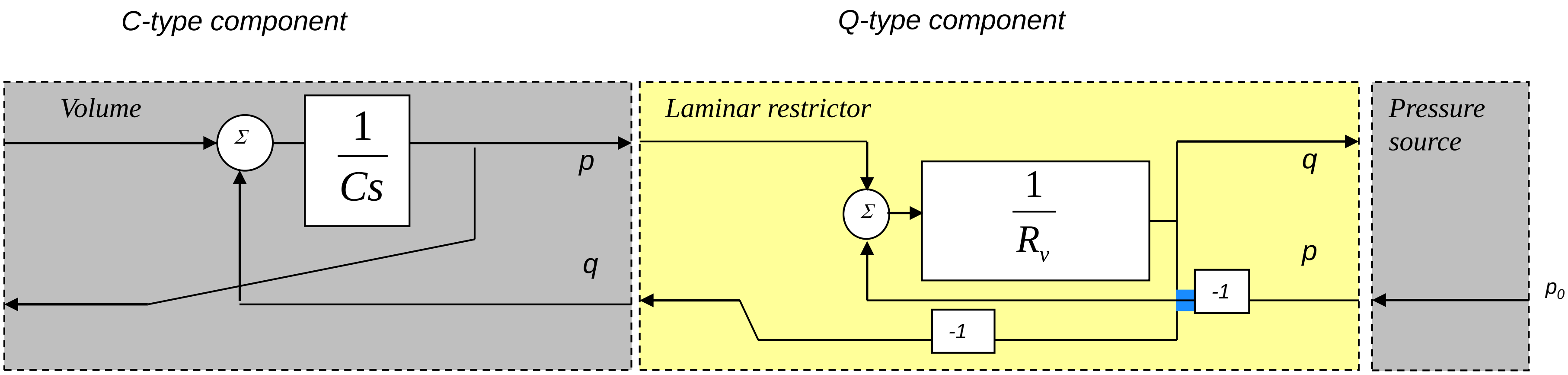
Blockdiagram of pressure source



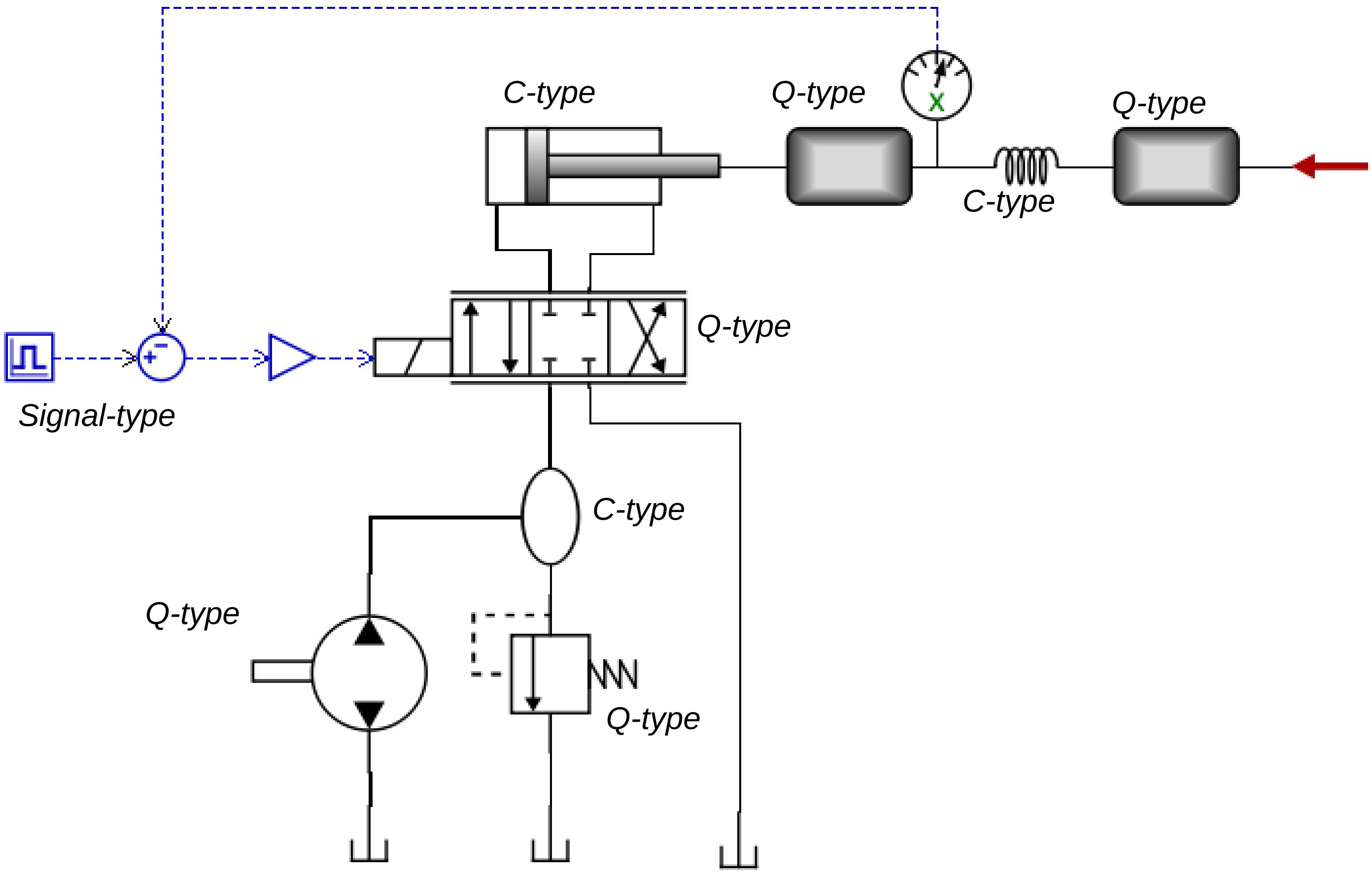
Blockdiagram of orifice connected to a line and a pressure source



Blockdiagram of orifice connected to a line and a pressure source (lumped parameters)

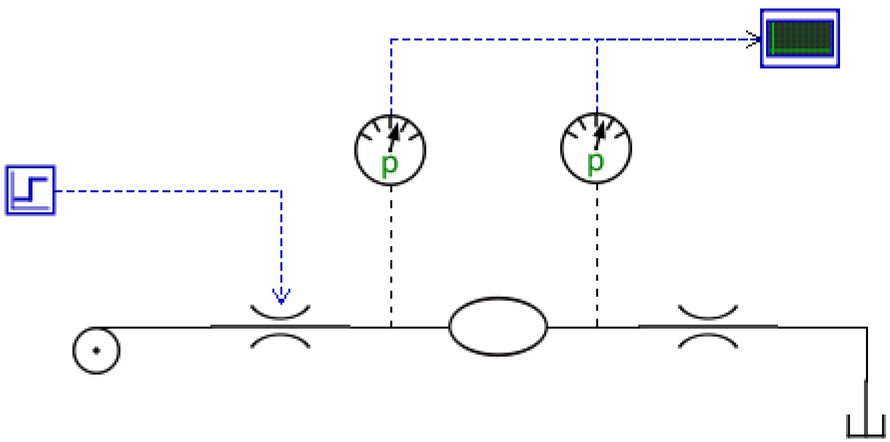


Example

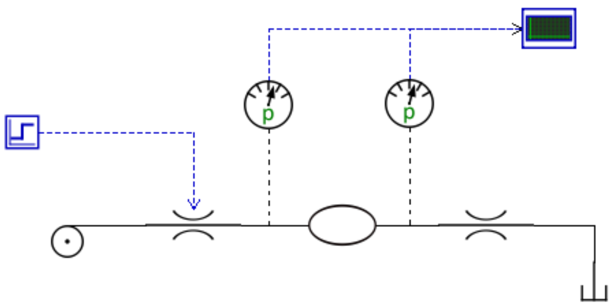


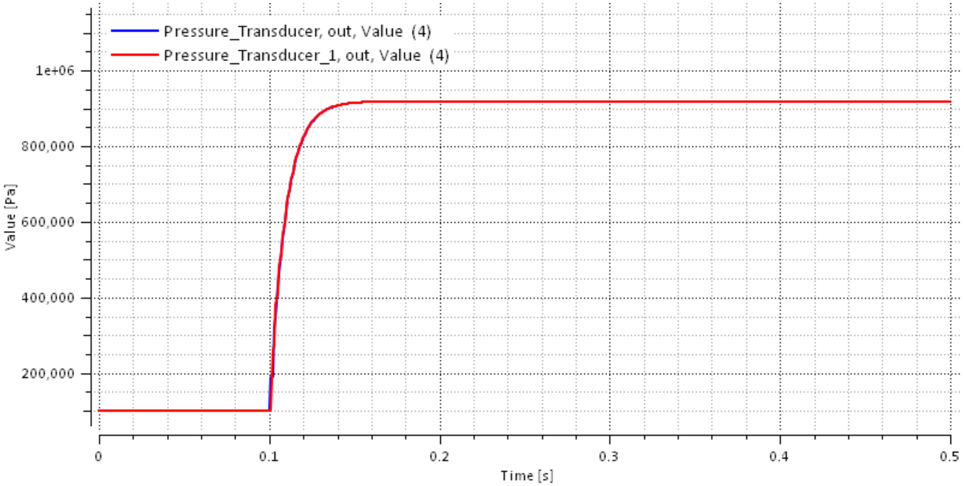
C-type
Q-type
Q-type
C-type
Q-type
Signal-type
C-type
Q-type
Q-type

Example

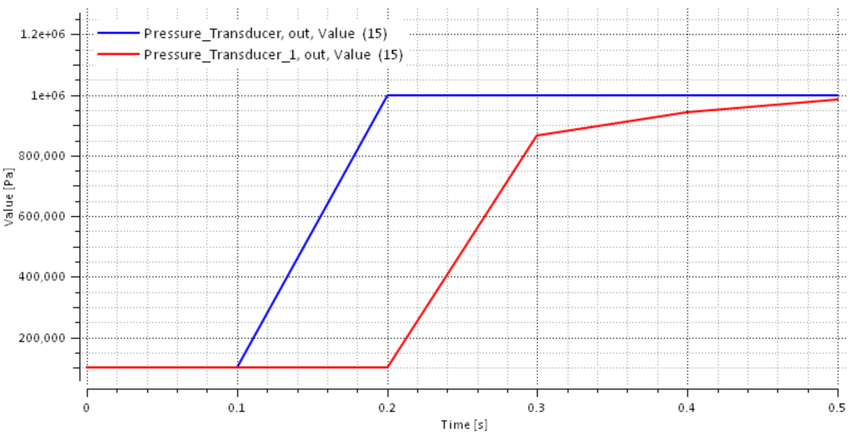


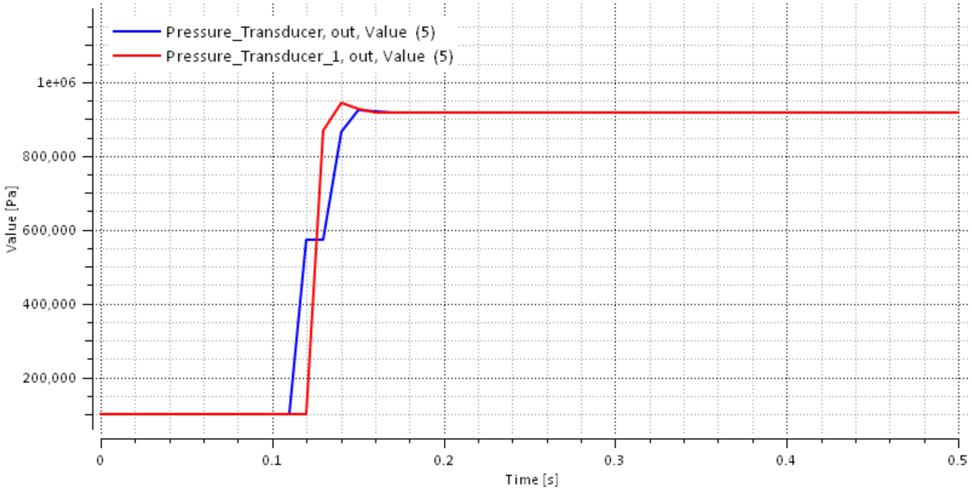
Example





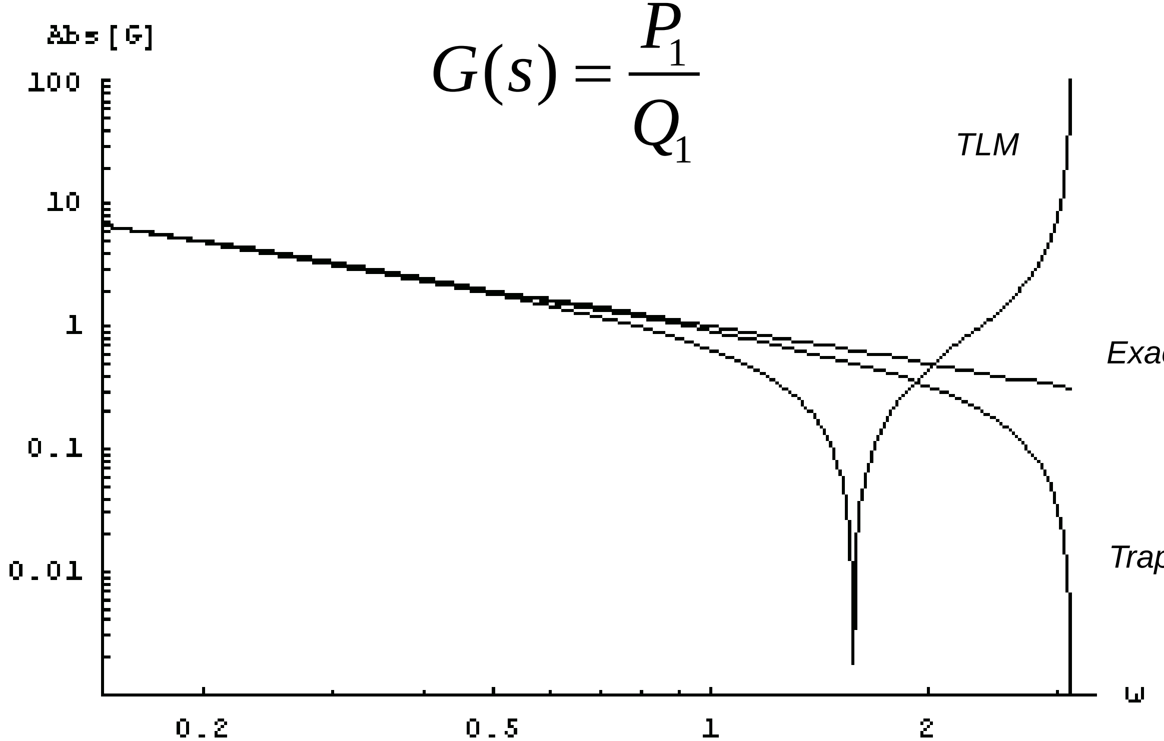
Pressure with times step h=0.001, 0.01, 0.1 [s]



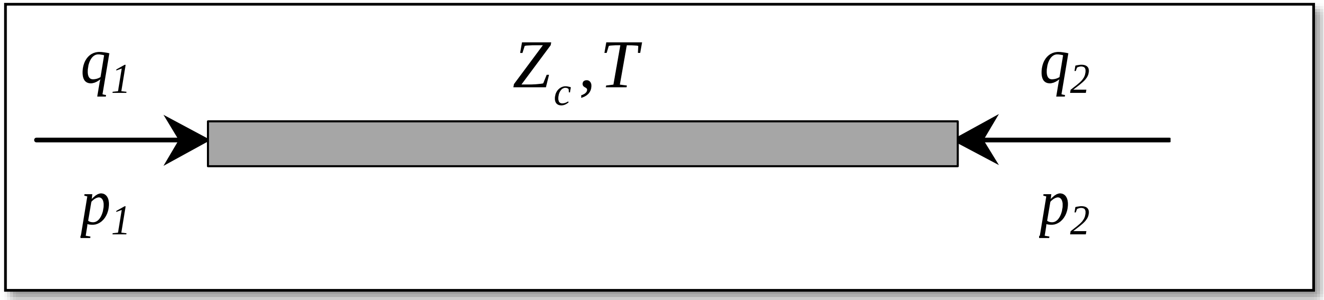


Bode diagram of TLM element with blocked end at 2 and the trapezoidal rule

$$G(s) = \frac{P_1}{Q_1}$$



Filtering of wave variable to supress high frequency oscillations



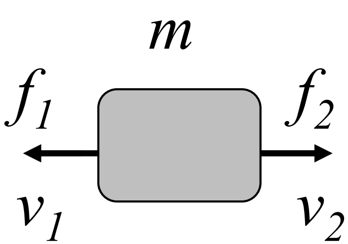
$$p_1(t) = c_1(t) + Z_c q_1(t)$$
$$p_2(t) = c_2(t) + Z_c q_2(t)$$

$$c_1(t) = p_2(t-T) + Z_c q_2(t-T)$$
$$c_2(t) = p_1(t-T) + Z_c q_1(t-T)$$

$$p_1(t) = c_{f1}(t) + Z_c q_1(t)$$
$$p_2(t) = c_{f2}(t) + Z_c q_2(t)$$

$$c_{f1}(t) = \alpha c_{f1}(t-T) + (1-\alpha) c_1(t)$$
$$c_{f2}(t) = \alpha c_{f2}(t-T) + (1-\alpha) c_2(t)$$
$$\alpha \approx [0.05, 0.5]$$

Mass with transmission line boundaries



$$\dot{v}_2 = \frac{f_1 - f_2}{m} - b\dot{v}_2$$

$$v_1 = -v_2$$

$$\dot{x}_2 = v_2$$

$$x_1 = -x_2$$

From transmission line

$$f_1 = c_{x1} + Z_{cx} v_1$$

$$f_2 = c_{x2} + Z_{cx} v_2$$

Solved for f1 and f2

$$\dot{v}_2 = \frac{c_{x1} - c_{x2}}{m} - (b + Z_{c1} + Z_{c2}) v_2$$

$$v_1 = -v_2$$

$$x_2 = \frac{v_2}{s}$$

$$x_1 = -x_2$$

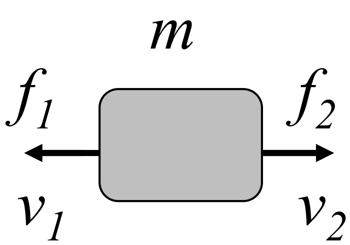
$$f_1 = c_{x1} + Z_{cx} v_1$$

$$f_2 = c_{x2} + Z_{cx} v_2$$

Mass with transmission line boundaries

Can be solved using biliear transform

$$s \rightarrow \frac{2}{h}\frac{1-q^{-1}}{1+q^{-1}}$$



$$V_2 = \frac{F_1 - F_2}{ms + b}$$

$$V_1 = -V_2$$

$$X_2 = \frac{V_2}{s}$$

$$X_1 = -X_2$$

From transmission line

$$F_1 = C_{x1} + Z_{cx} V_1$$

$$F_2 = C_{x2} + Z_{cx} V_2$$

Solved for F1 and F2

$$V_2 = \frac{C_{x1} - C_{x2}}{ms + b + Z_{c1} + Z_{c2}}$$

$$V_1 = -V_2$$

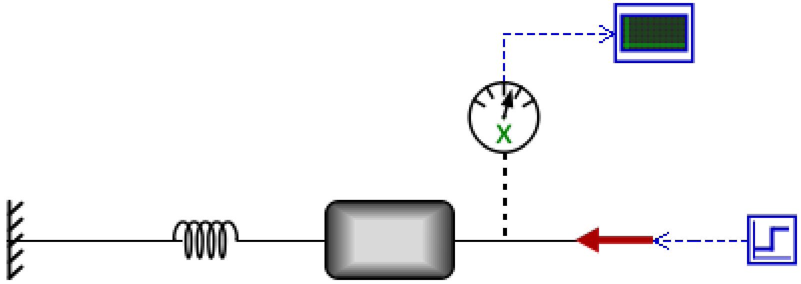
$$X_2 = \frac{V_2}{s}$$

$$X_1 = -X_2$$

$$F_1 = C_{x1} + Z_{cx} V_1$$

$$F_2 = C_{x2} + Z_{cx} V_2$$

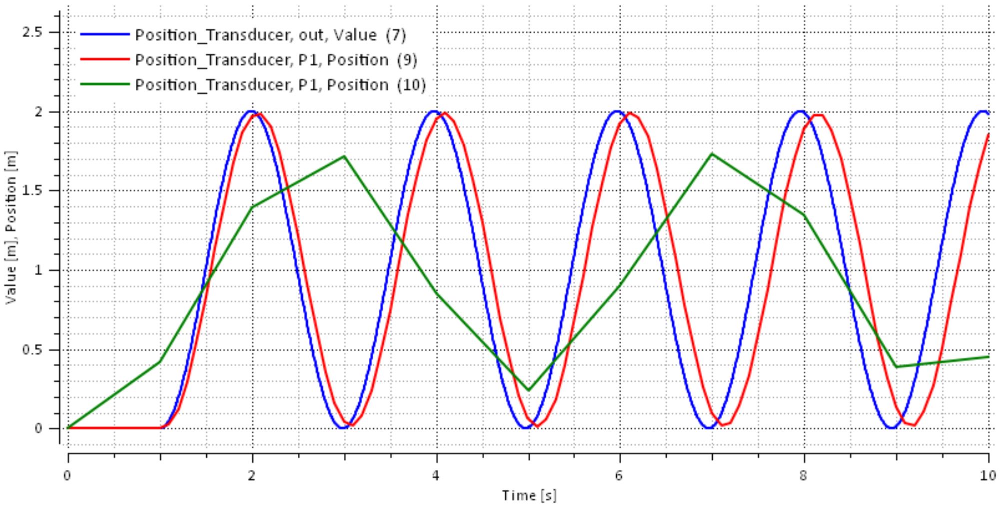
Mass Spring System



Parameters

$m = 100$ [kg]

$k = 1000$ [N/m]



Parasitic mass

$m_p = kh^2$

$m_p = 1000 \times 0.001^2 = 0.001$ [kg]

$m_p = 1000 \times 0.1^2 = 10$ [kg]

$m_p = 1000 \times 1^2 = 1000$ [kg]

Position with times step h=0.001, 0.1, 1 [s]

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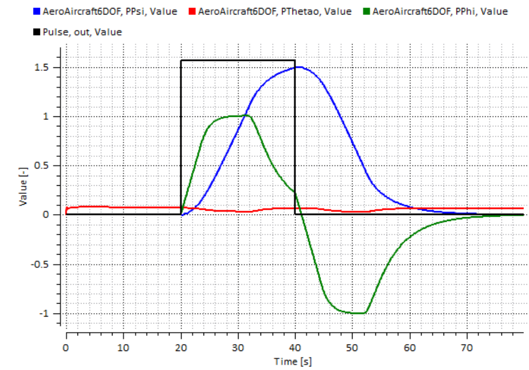
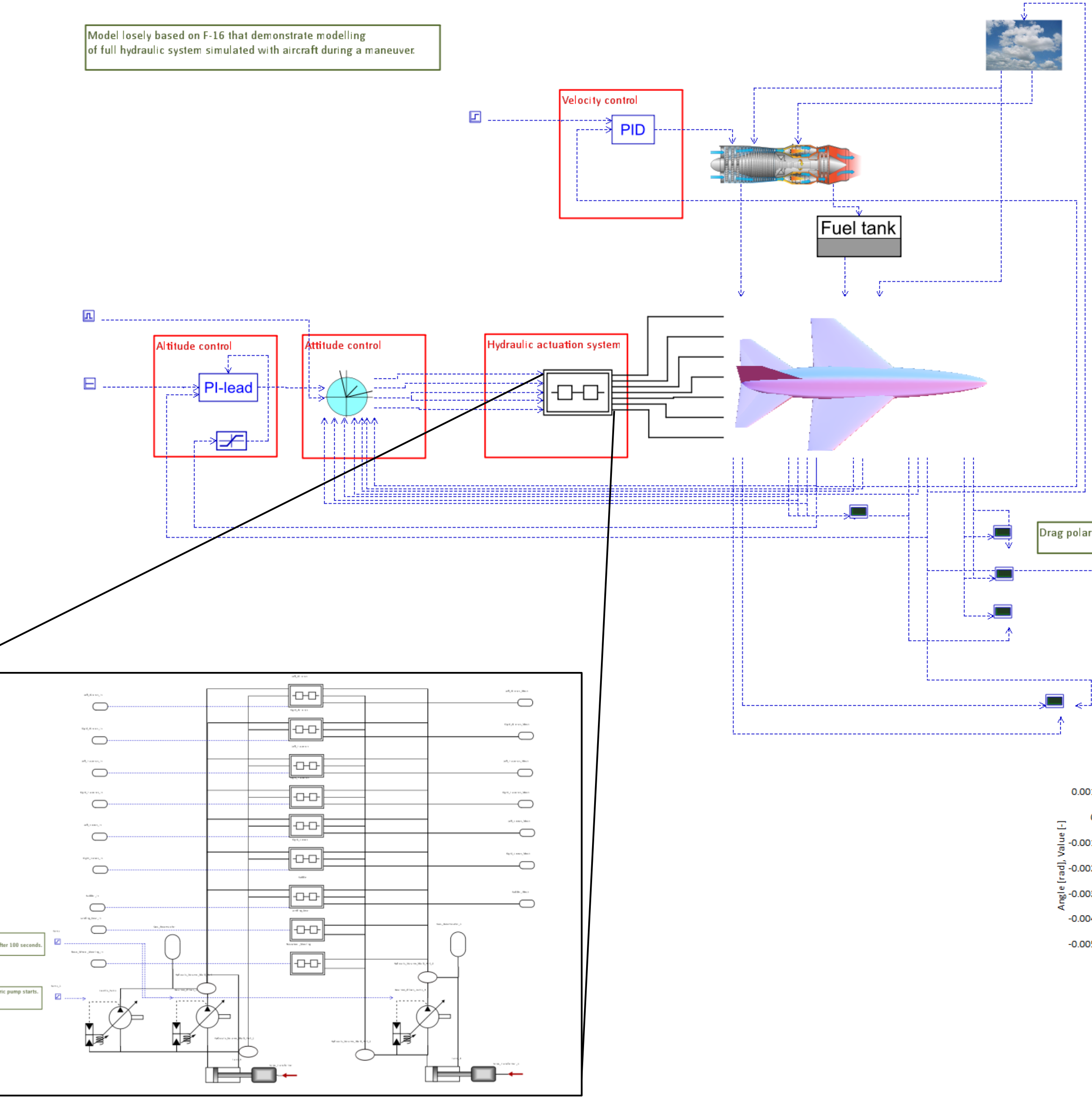
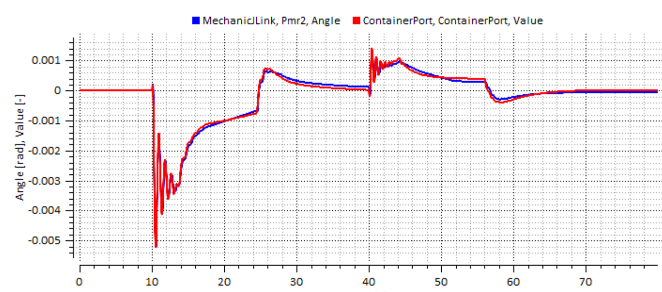
Integrated Aircraft System Simulation

Model losely based on F-16 that demonstrate modelling
of full hydraulic system simulated with aircraft during a maneuver.







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