Explicit pressure drop equations for Herschel-Bulkley fluid flow in pipes

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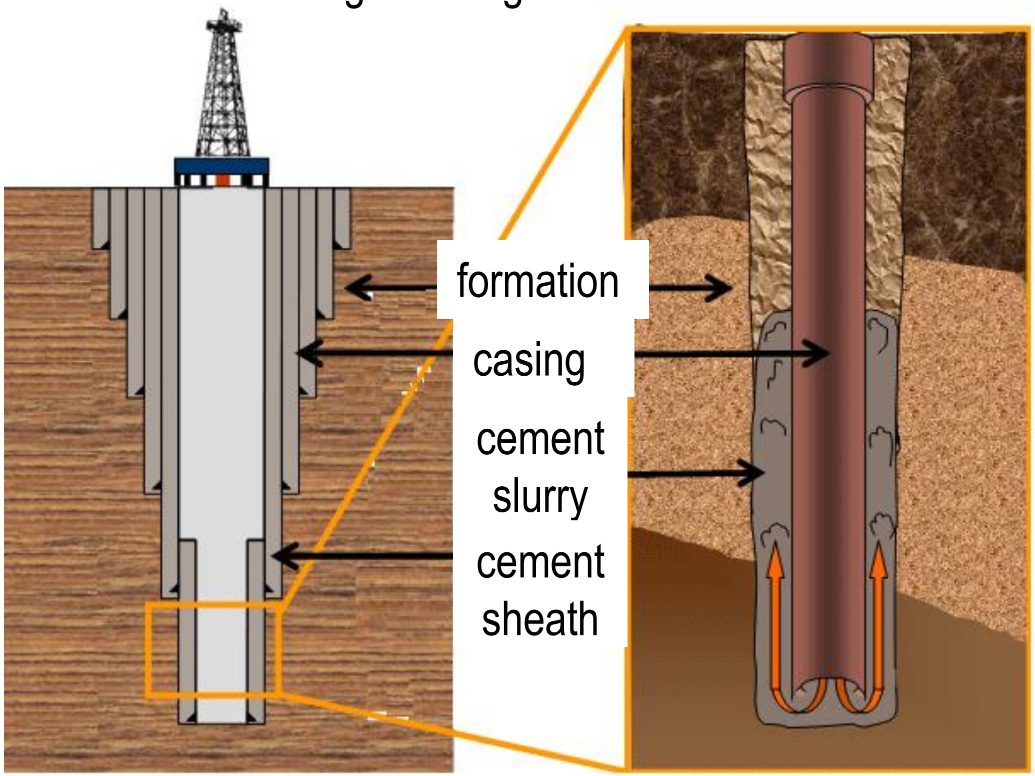
Background (1/2)

The flow of non-Newtonian fluids in pipes and conduits occurs in many situations including biology, the food processing industry and oil field services.

Cementing a casing into a well



Cement additive metering system

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Background (2/2)

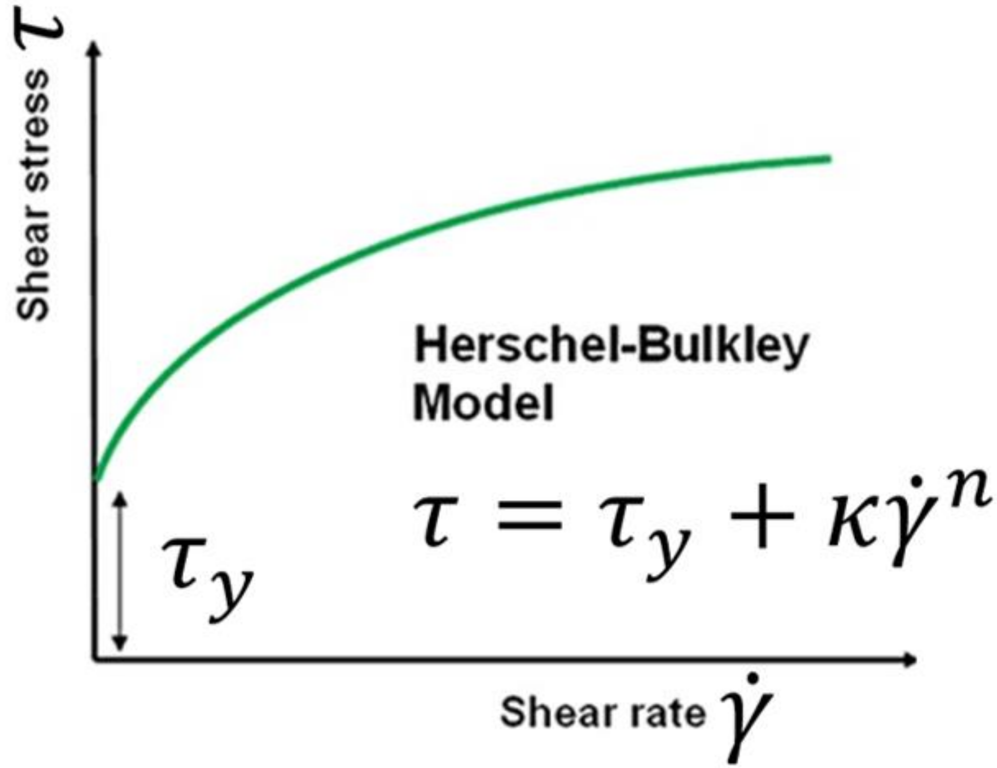
The stress-strain rate relationships for viscoplastic fluids are often modelled using the Herschel – Bulkley constitutive law.

$$\tau = \tau_y + \kappa\dot{\gamma}^n$$

κ consistency

τ_y yield stress

n flow index



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Outline

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- Flow rate in pipes given the pressure drop
- Pressure drop in pipes given the flow rate
- Modelica model
- Regularization
- Fluid properties and pipe dimensions
- Test cases
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  - Dymola translation statistics and simulation logs
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Introduction

Jahangiri et al. [1] described the modeling of non-Newtonian fluids characterized by the Power – law model and the implementation in Modelica.

In the present work we study the more general case of a Herschel – Bulkley fluid, including the effect of yield stress. We will restrict the presentation to the laminar flow regime, isothermal and incompressible. We will consider the choice of form of the pressure drop - flow rate relationship.

[1] Jahangiri, R. Streblow and D. Müller, Simulation of Non-Newtonian Fluids using Modelica, Proceedings of the 9th International Modelica Conference, September 3-5, 2012, Munich, Germany, DOI 10.3384/ecp1207657

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Flow rate in pipes given the pressure drop

Given the pressure drop, Δp, the mean velocity in the pipe , V, for a Herschel – Bulkley fluid can be determined from

$$\begin{cases} V = \frac{Dn}{2}\left(\frac{\tau_y}{\kappa}\right)^{\frac{1}{n}} \frac{(1-X)^{1+\frac{1}{n}}}{X^{\frac{1}{n}}}\left[\frac{(1-X)^2}{1+3n} + \frac{2X(1-X)}{1+2n} + \frac{X^2}{1+n}\right] & \text{if } X = \frac{4L\tau_y}{D\Delta p} < 1 \\ V = 0 & \text{otherwise} \end{cases} \quad (\Delta p \text{ to } V)$$

The pipe diameter is D and the length is L .

Given the mean velocity in the pipe, the equation can provide the pressure drop, but it is implicit and requires iterations.

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Pressure drop in pipes given the flow rate (1/3)

Introducing the Fanning the friction factor $f = \frac{D\Delta p}{2\rho V^2 L}$.

There are two alternative approximations:

Cruz, Coelho and Alves [2]	Swamee and Aggarwal [3]
based on simplifying physics assumptions	based on curve fitting to the exact solution
$f_{\text{CCA}} = \frac{16}{\rho V D}\left(\frac{1+3n'}{4n'}\right)^{n'} \mu_{\text{eff}}$ (V to Δp)$_1$ $n' = n\frac{\kappa\dot{\gamma}^n}{\tau_y + \kappa\dot{\gamma}^n}$ $\dot{\gamma} = \frac{8V}{D}$ $\mu_{\text{eff}} = \tau_y\dot{\gamma}^{-1} + \kappa\dot{\gamma}^{n-1}$	$f_{\text{SA}} = \frac{16}{Re_n}\left(1 + \left[\frac{1}{\left(36+\left(\frac{1.5}{n}\right)^{2.46}\right)^{0.5}}\frac{He}{Re_n}\right]^{\frac{0.958n}{2-n}}\right)$ (V to Δp)$_2$ $Re_n = \frac{\rho V^{2-n} D^n}{8^{n-1}\kappa}\left(\frac{4n}{1+3n}\right)^n$ $He = \frac{D^2\rho}{\tau_y}\left(\frac{\tau_y}{\kappa}\right)^{\frac{2}{n}}$

[2] D. A. Cruz, P. M. Coelho and M. A. Alves, A Simplified Method for Calculating Heat Transfer Coefficients and Friction Factors in Laminar Pipe Flow of Non – Newtonian Fluids, Journal of Heat Transfer, Vol 134, September 2012
[3] P. K. Swamee and N. Aggarwal, ‘Explicit Equations for Laminar Flow of Herschel – Bulkley Fluids’, The Canadian Journal of Chemical Engineering, Volume 89, December 2011, pp 1426-1433

Pressure drop in pipes given the flow rate (2/3)

In the limit of creep flow (high values of He/Re_n) it is found that equation (V to Δp)$_2$ under-predicts f. Since the value of f should always be superior or equal to $f_{\text{creep}} = \frac{2\tau_y}{\rho V^2}$, the equation (V to Δp)$_2$ can be improved by the operation

$$f_{\text{SA}}' = \max\{f_{\text{SA}}, f_{\text{creep}}\} \qquad \text{(V to Δp)}_3$$

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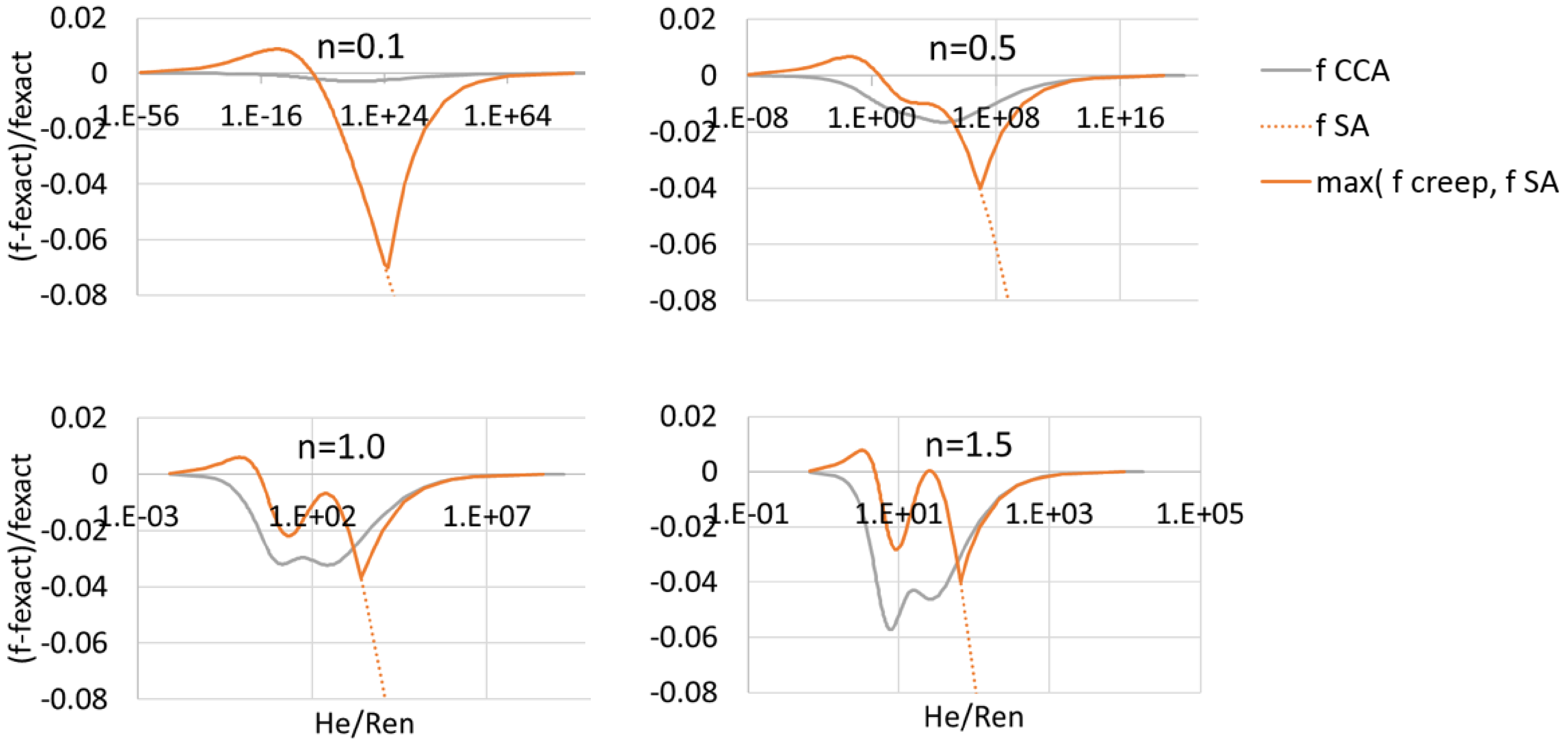
Pressure drop in pipes given the flow rate (3/3)

The relative error between the approximate relationships f_{CCA}, f_{SA} and f'_{SA} and the exact solution f_{exact} from (Δp to V) are shown below for a range of n values.



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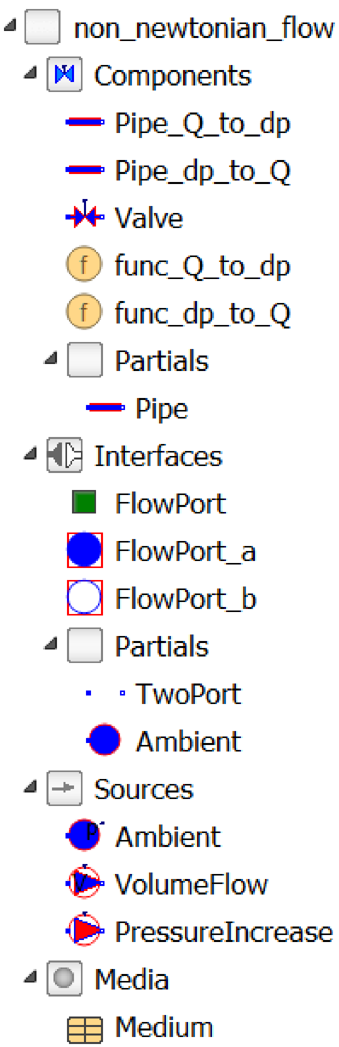
Modelica model (1/2)

The MSL Thermal FluidHeatFlow library was specifically adapted



Isothermal, incompressible and laminar flow of Herschel – Bulkley fluids

Connector variables `p` and `m_flow`

TwoPort partial flow element model to conserve mass flow and define the derived variables; the pressure difference `dp` and the volume flow rate `Q`

Medium record for the fluid properties `rho` , `consistency` , `cn` and `tau_y`

Pipe partial model to combine friction and hydrostatic pressure drops

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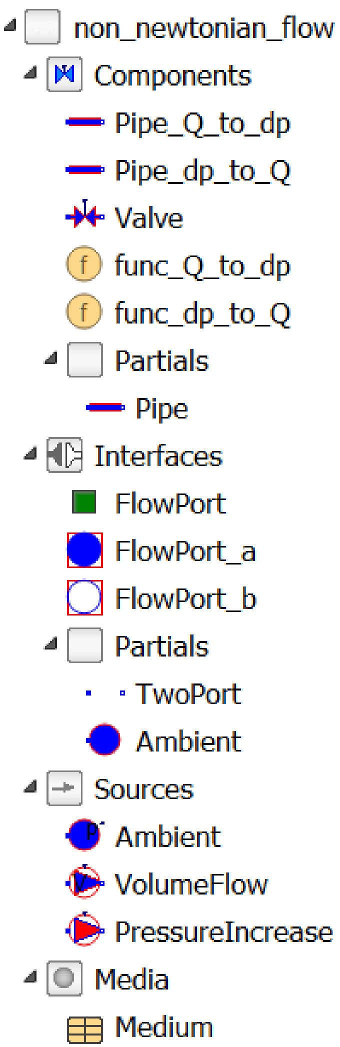
Modelica model (2/2)



Two types of pipe components are available –

Pipe_Q_to_dp uses the function func_Q_to_dp based on $(V \text{ to } \Delta p)_1$

```
model Pipe_Q_to_dp "Pipe - pressure drop from flow rate"
  extends Components.Partials.Pipe;
equation
  dp_fric=func_Q_to_dp(medium.tau_y,medium.consistency,medium.cn,len,d,dp_creep,small_velocity,small_pressure,Q);
end Pipe_Q_to_dp;
```

Pipe_dp_to_Q uses the function func_dp_to_Q based on $(\Delta p \text{ to } V)$

```
model Pipe_dp_to_Q "Pipe - flow rate from pressure drop"
  extends Components.Partials.Pipe;
equation
  Q =func_dp_to_Q(medium.tau_y,medium.consistency,medium.cn,len,d,dp_creep,small_velocity,small_pressure,dp_fric);
end Pipe_dp_to_Q;
```

Regularization (1/2)

In order to protect against numerical issues around the yield conditions, regularization of the pressure drop characteristic functions was carried out for the two alternative pipe elements

Pipe_Q_to_dp elements

For the purpose of the calcuation of frictional pressure drop, the magnitude of the mean velocity is never allowed to drop below the value of small_velocity

func_Q_to_dp

```
function func_Q_to_dp
  input . . . ;
  output Modelica.SIunits.Pressure dp_fric;
protected
  Real gamma, mueff, cndash;
algorithm
gamma:= 8*(abs(Q*4./Modelica.Constants.pi/d^2) +small_velocity)/d;
mueff:= tau_y*gamma^(-1) + consistency*gamma^(cn - 1);
cndash:= cn*consistency*gamma^cn/(tau_y + consistency*gamma^cn);
dp_fric:=32*Q*4./Modelica.Constants.pi/d^2*len/d^2*((1 + 3*cndash)/4/cnd
ash)^cndash*mueff;
annotation(LateInline=true,inverse(Q= func_dp_to_Q( . . . )));
end func_Q_to_dp;
```


Note the inverse function definition in the annotation

Regularization (2/2)

Pipe_dp_to_Q elements

The magnitude of the frictional pressure drop is never allowed to drop below small_pressure. If the pressure drop is below the yield value, the mean velocity is calculated from small_velocity/X.

func_dp_to_Q

```
function func_dp_to_Q
  input . . . ;
  output Modelica.SIunits.VolumeFlowRate Q;
protected
  Real X;
algorithm
X:= dp_creep/(abs(dp_fric) + small_pressure);
Q:=sign(dp_fric)*Modelica.Constants.pi*d^2/4*(if (X<1) then d*cn/2*(tau_
y/consistency)^(1/cn)*(1 - X)^(1 + 1/cn)/X^(1/cn)*((1 -
 X)^2/(1 + 3*cn) + 2*X*(1 - X)/(1 + 2*cn) + X^2/(1 + cn))
+small_velocity else small_velocity/X);
annotation(LateInline=true,inverse(dp_fric = func_Q_to_dp(  . . . )));
end func_dp_to_Q;
```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Fluid properties and pipe dimensions

The properties of the Herschel – Bulkley fluid studied and pipe element geometry are given in the table below

Internal pipe diameter D	0.05m
Pipe length	10m
Pipe orientation	horizontal
Consistency κ	3 Pa.s^n
Yield stress τ_y	10 Pa
Flow index n	0.5

The pressure drop required to overcome the yield stress is 8000 Pa, as shown below:

$$\Delta p_{\text{creep}} = \frac{4L\tau_y}{D} = \frac{4 \cdot 10 \cdot 10}{0.05} = 8000 \text{ Pa}$$

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Test cases (1/2)

For pure series and parallel networks comprising N pipe elements

	Outer boundary condition	Pipe element type	Size of non-linear system of equations
Series	Pressure	Pipe_Q_to_dp	{1)
		Pipe_dp_to_Q	{N}
	Flow rate	Pipe_Q_to_dp	{0}
		Pipe_dp_to_Q	{0}
Parallel	Pressure	Pipe_Q_to_dp	{0}
		Pipe_dp_to_Q	{0}
	Flow rate	Pipe_Q_to_dp	{N}
		Pipe_dp_to_Q	{1}

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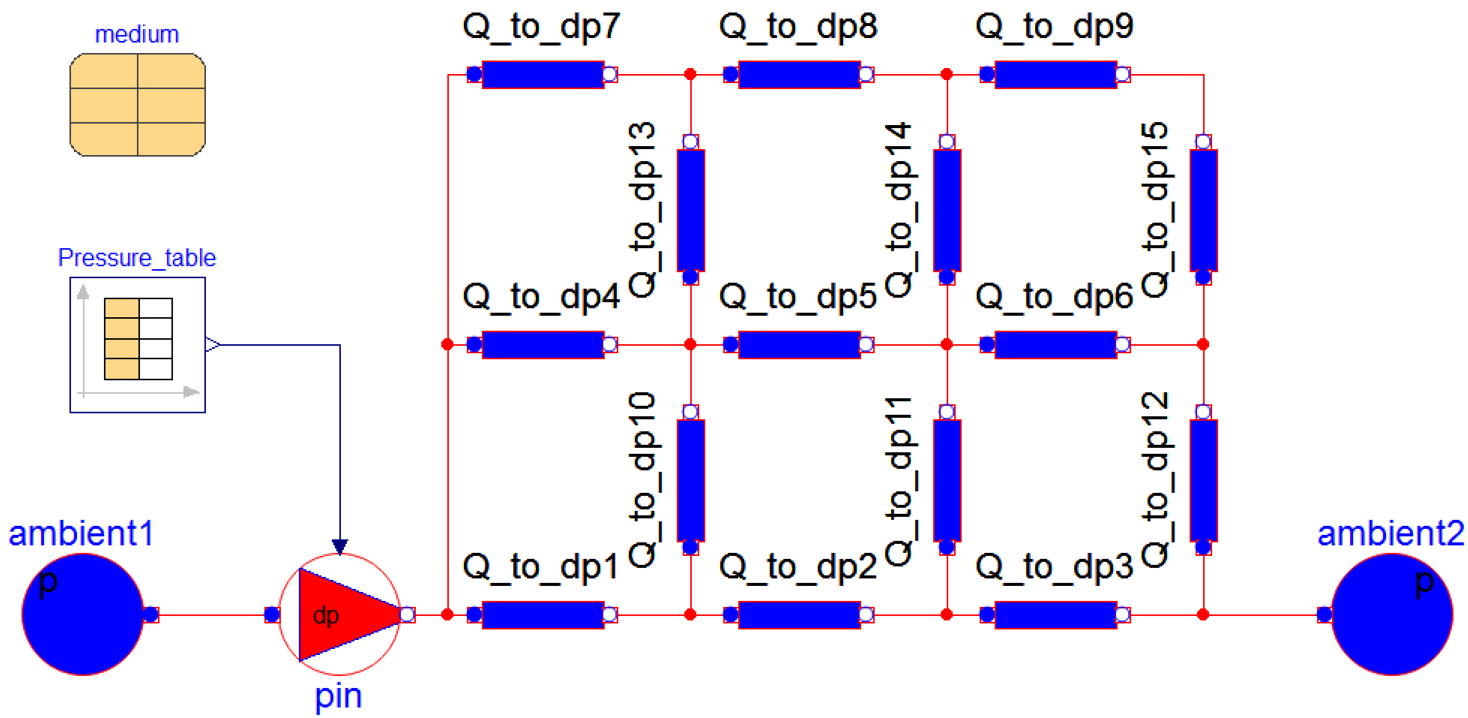
Test cases (2/2)

Hydraulic pipe networks were tested with the two alternative pipe elements and with two external boundary condition types.



	Outer boundary condition	Pipe element type
Test 1	Pressure	Pipe_Q_to_dp
Test 2		Pipe_dp_to_Q
Test 3	Flow rate	Pipe_Q_to_dp
Test 4		Pipe_dp_to_Q

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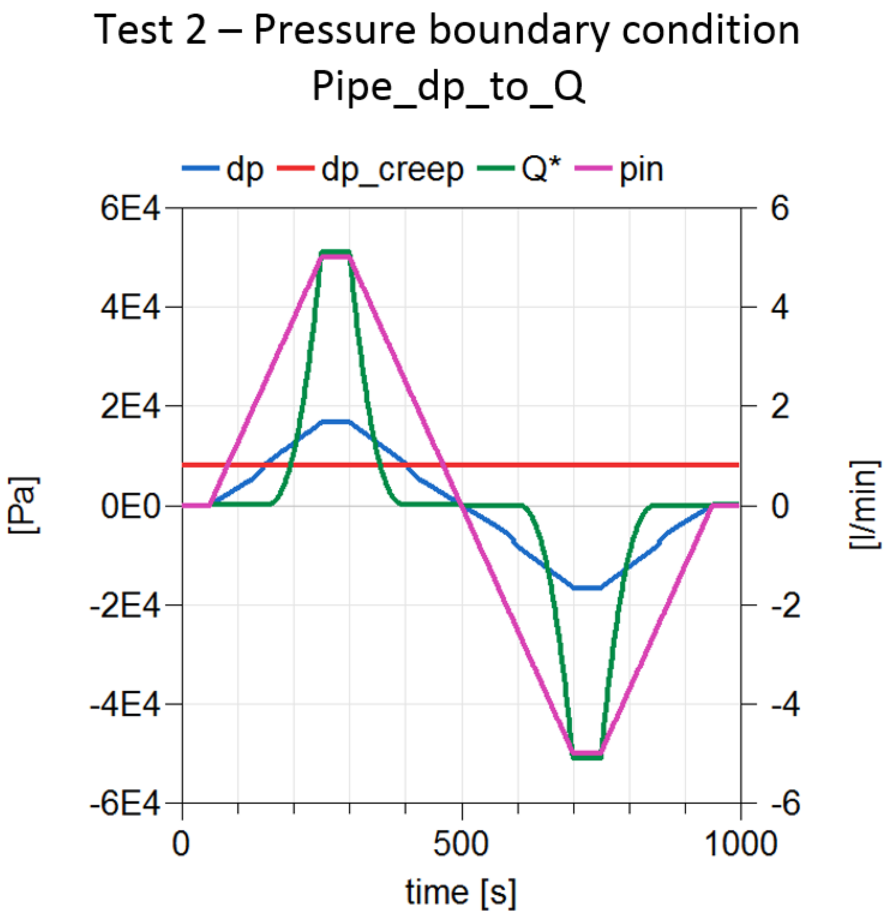
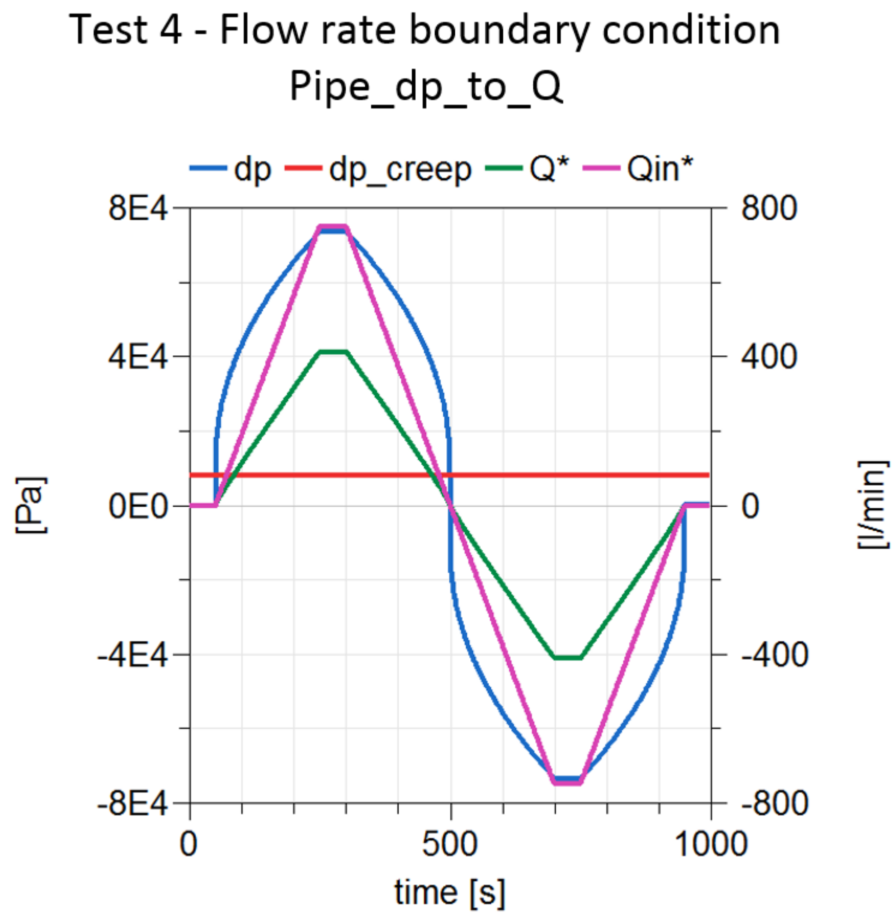
Dymola results

Plotting flow behavior in pipe elements 1





Tests 1 and 2 provide almost identical results. Same for Tests 3 and 4.

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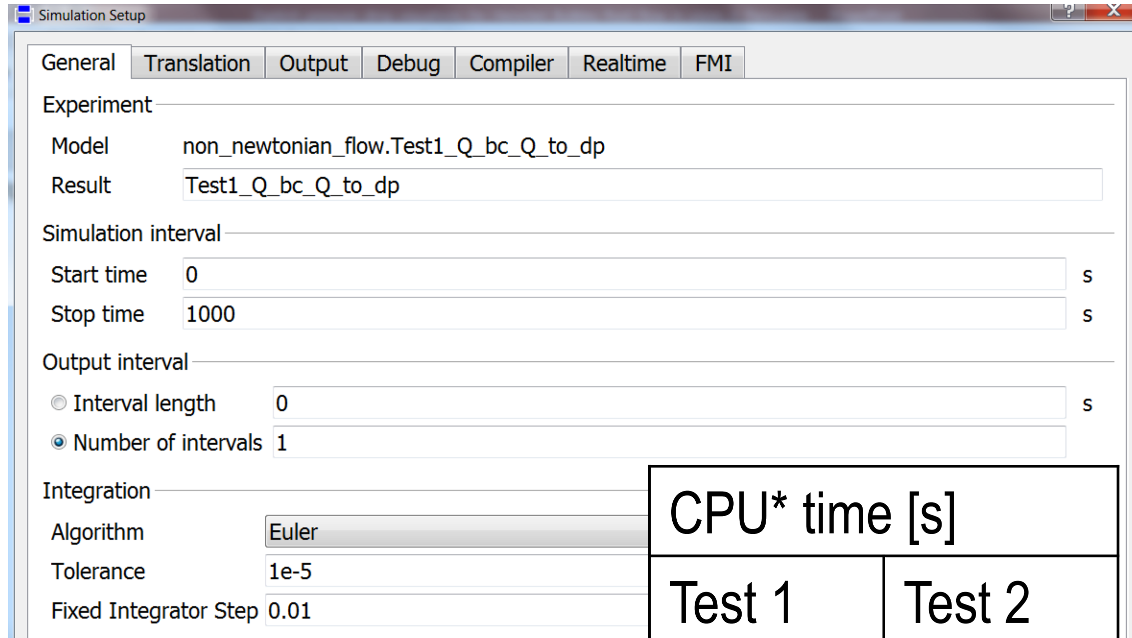
Dymola translation statistics and simulation logs

For the purpose of measuring CPU time, the integrator is set to fixed time step implicit Euler, δt=0.01s

Nonlinear system size after translation	
Test 1 {14}	Test 2 {15}
Test 3 {14}	Test 4 {15}



CPU* time [s]	
Test 1 8.68	Test 2 6.43
Test 3 12.60	Test 4 9.98

*CPU times are for single processor Intel Core i7-4900MQ 2.80 GHz

Insignificant file writing time by setting Number of intervals = 1

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Conclusions

1. Approximate explicit relationships to express laminar pressure drop of the flow of a Herschel – Bulkley fluid as a function of velocity have been tested and shown to produce results with sufficient precision for most engineering purposes.

2. The implementation in Modelica of the two alternative pipe models, based on (Q to Δp) and (Δp to Q) relationships, is described including regularization methods.

3. Model translation and simulation performance were monitored in Dymola.
   - For pure series networks of pipe elements with pressure type outer boundary condition, the formulation based on (Q to Δp) is preferred.
   - Conversely, for pure parallel networks of pipe elements with flow rate type outer boundary condition, the formulation based on (Δp to Q) is preferred.
   - For general networks both element formulations provide approximately equivalent performance.

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